On the intensity and loudness of sound

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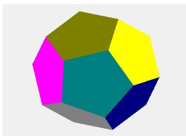

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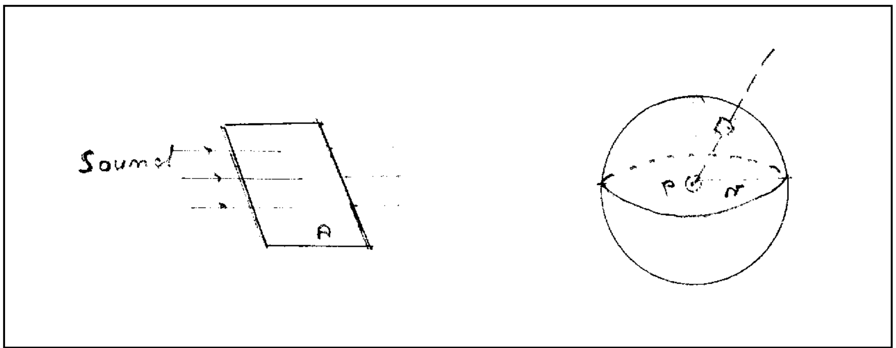
1. Power, intensity and loudness of sound

Sound is a wave phenomenon. Sound carry energy and momentum, but not material particles along its direction of propagation.
The energy that a wave source emits per second is called the *power* of the source, and is denoted *P*.
According to the definition of power:

$$P = \frac{\Delta E}{\Delta t}$$



Sound is waves, and a source of sound has a certain power, for example a small home loud speaker, has a label 20 *W* up to 100 *W* for a powerful speaker.
The applied mark is, however, not in compliance with the power of which power is emitted from the speaker, since the power mark of the speaker refers to the electrical power consume of the speaker, and there is always a loss of power in the electrical wires together with the mechanical parts of the speaker.
The intensity *I* of sound at a certain point of space is defined by the power, which passes through an area *A* placed perpendicular to the propagation of the sound, divided by the area *A*.

$$I = \frac{P}{A}$$ The intensity is measured in W/m^2

According to this definition, the intensity is (for smaller areas) independent of the size of the area, since the power that passes the area is proportional to that area.

The intensity is the physical measure of how powerful the sound is. The human perception of sound is, however, quite another matter, since it does not follow an absolute scale, but rather a logarithmic scale. This roughly means that a ten doubles of the intensity is perceived as an increase of 10.
For that reason one usually declare the loudness of sound not in W/m^2, but in the measure decibel (dB).
1 dB = 1/10 B (named after the inventor of the telephone Alexander Graham Bell). The decibel scale is, however, a logarithmic scale, which has a reference point 0 dB at $I_0 = 10^{-12}$ W/m^2, which is also the hearing threshold (The least intensity a normal ear may perceive). The strength of the sound is then denoted by the loudness and is measured in dB, although dB is not a physical unit.

If *L* denotes the loudness in dB, and *I* is the intensity, then they are connected by the formula:

$$L = 10\log\frac{I}{I_0}$$

If $I = I_0$, then $L = 0$.
When the intensity is 0.2 W/m^2 it corresponds to a loudness:

$$L = 10\log\frac{0,2}{10^{-12}} = 113 \ \ dB\,,$$

which is a rather powerful volume.

A whisper is about 30 dB. Sound becomes painful at about 135 dB, and the strength is therefore denoted as the pain threshold.
The consequence of the logarithmic scale is that, when the intensity is increased by a factor 10 then the volume measured in dB is increased by the amount10 dB. If the intensity is increased by a factor 100, then the volume measured in dB is increased by the amount 20 dB.
This follows from the definition of volume in db.

$$L_{10} = 10\log\frac{10I}{I_0} = 10\log 10 + 10\log\frac{I}{I_0} = 10\log\frac{I}{I_0} + 10$$
$$L_{100} = 10\log\frac{100I}{I_0} = 10\log 100 + 10\log\frac{I}{I_0} = 10\log\frac{I}{I_0} + 20$$

2. The inverse square distance relation for propagation of waves.

If you enclose a sound source at the centre of a mathematical sphere, assuming that the sound is emitted isotropic (the same in all directions), then the intensity at the surface of the sphere $I(r)$, must be overall the same, and if there is no loss of power, then the power that passes through the surface of the sphere must be the same as the power emitted from the source.

The relation between the power that passes through an area and the intensity at that area is:

$$I(r) = \frac{P_A}{A} \quad \Leftrightarrow \quad P_A = I(r)A$$

If we choose the area A as the surface of the sphere, we have $A = 4\pi r^2$, we find:

$$I(r)4\pi r^2 = P \qquad \text{or} \qquad I(r) = \frac{P}{4\pi r^2}$$

The last equation is called *the inverse square distance law*, since it states that the intensity is inversely proportional to the distance from the source to the point of receiving.

If the distance is doubled, then the intensity is reduced to one fourth.

Using this formula, you may calculate a lot of things, for example estimate the loudness at a rock concert, with 4000 W loudspeakers, and where a person is placed at a distance of 10 meters.

First we calculate the intensity from $I(r) = \frac{P}{4\pi r^2}$, which gives $I = \frac{4000W}{4\pi(10m^2)} = 3.18\, W/m^2$, and then the loudness:

$$L = 10\log(\frac{3.18}{10^{-12}}) = 125\, dB$$

(Certainly a powerful volume, but still below the pain threshold).

We shall then derive a formula, which shows how much the loudness in dB is attenuated from a distance r_1 to a distance r_2. It shows up, that the answer is independent of the intensity.

$$L_1 - L_2 = 10\log(\frac{I_1}{I_0}) - 10\log(\frac{I_2}{I_0}) = 10\log(\frac{I_1}{I_2}) =$$

$$10\log\left(\frac{\frac{P}{4\pi r_1^2}}{\frac{P}{4\pi r_2^2}}\right) = 10\log(\frac{r_2^2}{r_1^2}) = 20\log(\frac{r_2}{r_1})$$

From the calculation rules for the logarithmic function we thus find.

$$L_1 - L_2 = 20\log(\frac{r_2}{r_1})$$

Using this simple formula one may calculate, how much the loudness of the sound is attenuated from the distance r_1 to a distance r_2 . For example will the loudness decrease with 20 dB, if you move from the distance 1.0 *m* to the distance 10 *m*. This follows from the formula above:

$$L_1 - L_2 = 20\log(\frac{r_2}{r_1}) = 20\log(\frac{10}{1}) = 20\, dB$$

In class the teacher may wonder, whether he may notice if two students whisper to each other on the last row, when they whisper with the loudness 30 dB at the distance of 25 cm from the ear. The last row is considered to be 7 meters from the teacher.

$$L_1 - L_2 = 20\log(\frac{r_2}{r_1}) = 20\log(\frac{7}{0.25}) = 29\, dB$$

Since 30 dB - 29 dB is on the hearing threshold the answer is no.