Calculating the runway for DC-7
and
The buoyancy on a DC-7 in the air

This is an article from my home page: www.olewitthansen.dk

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Contents

1. Calculating the runway before take off for a DC-7.


Photo ©: Nicholas A Vollaro (photos at airliners.net)

We shall first calculate the estimated runway before take off under the assumption that the engines deliver a constant power P. The calculation is actually exceedingly simple. From the definition:

(1.1) $\dfrac{ds}{dt} = v$ it follows $s = \int v dt$

Also from the theorem of mechanical work:

The work done by the resulting force on a mechanical system is equal to the change in kinetic energy.

(1.2) $$Pt = ½mv^2$$

It follows then that when P is constant: $Pdt = d(½mv^2) = mvdv$,

From which we get an expression for $dt = \dfrac{m}{P} vdv$, and if we insert in (1.1) we get:

(1.3) $$s = \int vdt = \frac{m}{P}\int v^2 dv \quad \Rightarrow \quad s = \frac{m}{3P}v^3$$

If the airplane has an initial velocity $v_0 = v(0)$ then:

(1.4) From $\dfrac{ds}{dt} = v$ it follows as before: $s - s_0 = \int_0^t vdt$.

Since $Pt = ½mv^2 - ½mv_0^{\ 2}$ it follows as before (when P is constant)

$$Pdt = d(½mv^2) = mvdv .$$

From this equation we get: $dt = \dfrac{m}{P} vdv$, which we insert in (1.4)

$$s - s_0 = \int_0^t vdt = \frac{m}{P}\int_{v_0}^{v} v^2 dv \quad \Rightarrow \quad s - s_0 = \frac{m}{3P}(v^3 - v_0^{\ 3}) .$$

We shall now try to confront this simple calculation with some data relating to the Douglas DC-7.

Empty weight: 33005 *kg*, Max takeoff weight: 64864 *kg*, Wing Area: 152.1 m^2, and
Engine Power (each) 2535 *kW*

We assume that take off takes place at a speed of 250 *km/h*.

Using (1.3) : $s = \frac{m}{3P}v^3$, and assuming an empty weight, we find the runway before take off:

$$s = \frac{3.3\,10^4\ kg}{3 \cdot 4 \cdot 2.553\,10^6 W}(250/3.6\ m/s)^3 = 361\ m$$

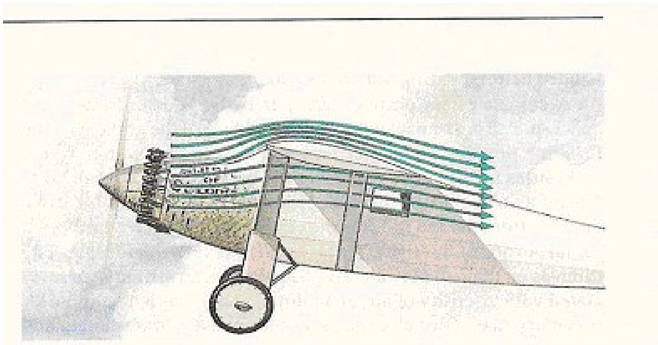
If we want the runway for the maximum weight, we must multiply by 64864/33005 to give 709 *m*

Since most runways are about 1500 *m,* the calculations make good sense, however, we have not taken account the loss of power due to the landing gears and the air drag. It is pure speculation to estimate the factor we should multiply with to take that into account. In fact an opposite air drag, will certainly contribute to an earlier take off, due to Bernouilli's law.

Of course one could make the same calculation on e.g. the Airbus A-380, or the Booing 777, and although they will require a larger runway, the results will probably not differ significantly.

2. The art of flying

Bernoulli's law can deliver only a qualitative explanation on many physical phenomenon in the real world. For example, why it is possible to fly (without barking with the wings), which still remains a mystery for many people outside the scientific community.



14–26 Flow lines around an airplane wing.

The figure demonstrates the principle of flying according to Bernoulli's law.
It shows the streamlines on a wing on an airplane.
Because of the shape of the wing, the air must travel a longer stretch above the wing than below the wing.
The air above the wing must therefore travel with a higher speed than below.

Following Bernoulli's law, it means that the pressure on the top side of the wing is diminished, compared to the pressure on the bottom side.
This difference in pressure causes the "buoyancy" counterpart to gravity, and that keeps the plane in the air.
The construction of airplanes is based on a hundred years of experience, engineering ingenuity and wind tunnel experiments. Aerodynamics is a very complex science, especially when it comes to turbulence, and it certainly does, when it concerns flying.

2.1 Example:

To illustrate the application of Bernoulli's law to flying, we shall make a simple (but unrealistic) calculation.

We assume that the speed of the air flow on the upper side of the wing is 20% larger than on the lower side.
We can then find from Bernouli's law: (Since there is no contribution from potential energy)

$$p_1 + ½\rho\, v^2 = p_2 + ½\, \rho\, (1.2v)^2 \;\; => \;\; \Delta p = p_1 - p_2 = ½\, \rho\, 0.44v^2.$$

Inserting $\rho = \rho_{air} = 1.29\ kg/m^3,\ v = 360\ km/h = 100\ m/s,$ we get:

$\Delta p = 2.84 \cdot 10^3\ N/m^2 = 2.84 \cdot 10^{-2}\ atm = 284\ kp/m^2$. (1 $atm. \approx 10^5\ N/m^2 \approx 1\ kp/cm^2$).

The DC-7 has a wing area of 152.1 m^2, and it corresponds therefore to a buoyancy of:

$$2.84\ 10^2\ kp/m^2\ 1521\ m^2 = 4.32\ 10^4\ kp.$$

This correspond roughly to the weight of 43.2 ton.

Bernouilli's law is based on a laminar flow, so this calculation cannot be other than a rough estimate, since turbulence probably plays an active role, and there exists no formal theory for turbulence.
However, the calculated buoyancy is qualitatively enough to keep the DC-7 flying, since the empty weight is 33,000 ton, and the max weight is 64,864 ton.