The physics of Radioactivity

The law of radioactive decay. Transformations of the atomic nucleus. Energy considerations on radioactive decays

Chapter 8 of the textbook
Elementary Physics 3

Ole Witt-Hansen

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Informative remark: The present chapter is a translation from the Danish textbook: Elementary Physics 3. However, the texts, when it appears in the figures are not translated. On the other hand the figures and the supplementing text should speak for themselves.

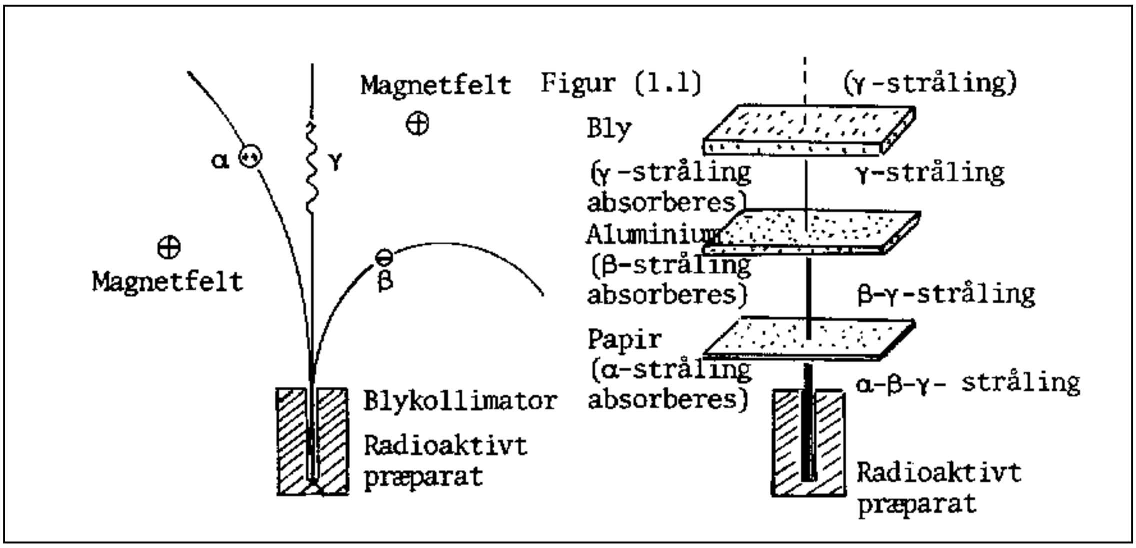
Contents

1. Natural radioactivity, α, β and γ – radiation

Already in 1896 the Frenchman Henri Becquerel discovered that earths containing uranium emitted a penetrating radiation, being able to blacken a photographic record or lighten a fluorescent screen. The married couple Marie and Pierre Curie posed the task to isolate the substances which caused the radiation. After a comprehensive chemical analysis they succeeded in 1898, from about one ton of earths to extract a few grams of two previously unknown, so called radioactive elements, which they gave the names *radium* and *polonium*.
Since then one has identified about 10 radioactive elements, of which the most well known is uranium (*U-9*2), which is fuel the in nuclear power plants.
All elements found in nature with atom number higher than *Pb*-82 are radioactive.

The discovery of radioactive substances opened a new chapter in physics. In particular Rutherford and his collaborators (Including Bohr) performed many experiments with radioactivity and from these experiment they were able to conclude that the radioactive radiation consisted of particle emitted from the atomic nucleus.
Radioactivity is then messages from the atomic nucleus, and by analyzing the radiation one has obtained considerably insight in the complex structure of the atomic nucleus.

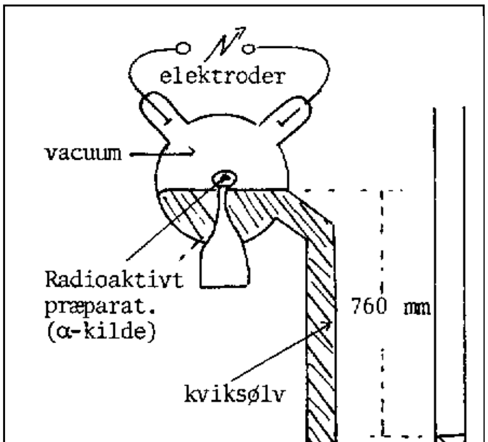
One of the earliest discoveries of Rutherford and his collaborators was that the radiation consisted of three components, which may be characterized either by their electrical properties or by their ability to penetrate materials.
The three components were named alpha-, beta- and gamma-radiation, which is usually written $\alpha-,\beta-,\gamma-$radiation.



By deflection in a magnetic field, they could show that the radiation included a positively charged component ($\alpha-$radiation), a negatively charged component ($\beta-$radiation), and a neutral component ($\gamma-$radiation).
Experiments showed furthermore that that the $\alpha-$particles could be stopped by a few millimetre of paper or few centimetre of air.
The $\beta-$radiation on the other hand could penetrate both paper and glass, but was stopped by a few millimetres of aluminium.

The $\gamma-$radiation, however, could penetrate almost anything, and it could only be stopped by several centimetres of lead (*Pb*).
By deflection in magnetic fields Rutherford and others found that the ratio between charge and mass *q/m* for the $\beta-$radiation was very close to, (but a little smaller) than *e/m* for the electron, and it was concluded (correctly) that the $\beta-$radiation was in fact composed of electrons.
(The deviation from earlier measurements of the *e/m* ratio can be explained from the theory of relativity, since the emitted electrons are relativistic, which gives a measurable increase in the mass)

Rutherford also measured the *q/m* ratio for the $\alpha-$particles, and it showed up that the ration suited with the ratio of the helium nuclei. Rutherford therefore assumed that the $\alpha-$particles were probably helium-nuclei. A presumption he later confirmed by an elegant experiment.



Figur 1.2

The figure shows the schematic setup of Rutherford's experiment. The $\alpha-$source is placed behind a thin wall of glass, so thin that it allows the $\alpha-$particles to pass into the larger glass tube, where they are collected, since they could not penetrate the glass wall of the tube.
On the other hand none of the γ- and β-particles would be stopped by the glass wall in the tube.
In the tube is high vacuum, (ingeniously and simple obtained by first filling the entire apparatus (upside down) with quicksilver, and then turn it around, and letting the quicksilver sink until the difference between the levels in the open and the closed tubes are 760 *mm*, the atmospheric pressure).
After a certain time, there are only *He*-atoms in the container, (and possibly a few *Ag*-atoms), since the $\alpha-$particles have acquired electrons from the glass wall.

In the (Mickey-Mouse) shaped container are melted in two electrodes, and if there is induced a high voltage on the electrodes a discharge through the tube will occur.
The stream of electrons between the electrodes will ionize the *He*-atoms, and they will subsequently emit light with the spectrum characteristic of the *He*-atom.

The experiment confirmed that the $\alpha-$particles are in fact *He*-nuclei, which was another of Rutherford's great achievements.

The $\beta-$particles had already been identified as electrons, and the $\gamma-$radiation, turned out to be very hard *X*-rays, having wavelengths in the interval (10^{-4} *pm* to 1 *pm*).

2. Methods to register radioactive radiation

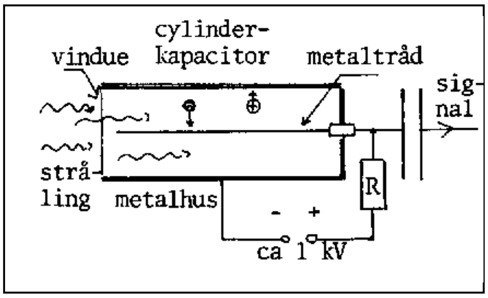
They can roughly be separated into four categories.

2.1 Dosimeters

Dosimeters are used to register an accumulated radiation, that is, to measure the radiation dose which is delivered in a certain place in a longer period of time. Earlier a dosimeter could simply be an encapsulated photographic record that you carried around with you, and thereafter it was induced to estimate the blackening. Dosimeters are particularly applied in laboratories, power plants and in health physics.

2.2 Devices (counters) to register immediate radiation

Among the oldest and well known devices is the Geiger-Mueller counter (*GM*-tube). (Nowadays it has been replaced by electronic devices). The *GM*-tube is in principle a cylinder-capacitor containing a gas at a very low pressure. See the figure (2.1) below.



In the end of the metal cylinder there is a thin mica window, where the radiation enters.
A charged particle which enters the tube will ionize the gas atoms in the tube and the electrons will thereafter accelerate towards the thin rod in the centre of the tube. On their way they can ionize more atoms.
This multiplicative ionization will result in a shower of electrons that will last until the positive ions have reach the outer cylinder.
This electric shower will give rise to a electric pulse that isaccumulated in the capacitor outside the *GM*-tube. The signal can then be amplified, and perhaps converted to an acoustic signal, as anyone has seen many times (tac, tac, tac).
The problem of stopping the ionizing in the *GM*-tube, is simply solved, since even with a very small current in the *GM*-tube, the voltage over the *GM*-tube will fall drastically because of the large pre-resistor (some $M\Omega$), and as the voltage diminishes, the ionization stops. The voltage is reloaded, and the *GM*-tube is ready for registering for a new particle.

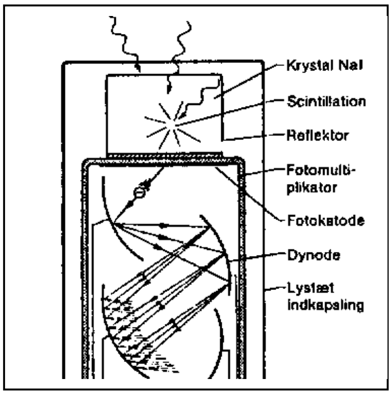
During the active period for the *GM*-tube, the tube is inactive for registering a particle for a certain time. This is called the dead-time.
Outside the dead time the *GM*-tube can register $\alpha -$ and $\beta -$particles almost 100%, whereas only 1% of the γ-particles are registered.
The γ-particles do not ionize the atoms directly, but they can nevertheless in a minor scale be detected, since they can cause photo electrons, Compton electrons, or pair-creation of electrons to be emitted from the atom, and thereby start a ionization chain, which results in pulse.

2.3. Detectors apt to measure the energies of radioactive radiation

For lower voltages over the *GM*-tube, the energies for $\alpha -$ and $\beta -$particles are roughly proportional to the number of ions created, and thus to the strength of the signal.
But it requires that the voltage over the *GM* –tube is suitable low, since then, the number of ions made by collisions-ionizations is proportional to the primary ions.
If the *GM*-tube is used in this manner, it is called a *proportional-counter* or an *ionization-chamber.*

In that case, however, the amplification becomes several thousand times less than if the tube is applied as a *GM*-tube, where the signal is independent of the number of primary ions.

Figure (2.2)

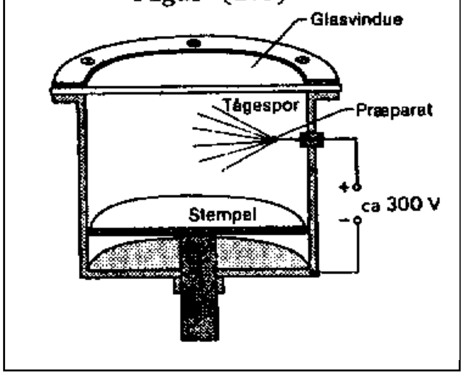


The scintillation counter is based on the fact that some substances exhibit fluorescence, which means that they emit a flash of light, when they are hit by a charged particle or a photon.
In general the strength of the flash of light is proportional to the energy of the primary particle.
If the signal thereafter is then amplified in a photo-multiplier, we have in principle a *scintillation counter*, being able to (in contrast to the *GM*-tube) to register the energies of radioactive particles.
As the scintillator, one may for example apply a *NaI* crystal, but nowadays (1980) semi-conductors are applied.These counters e.g. *Ge-Li* counters have been a great progress.
They have great efficiency (register the radiation almost 100%), and great resolution power of the energy. Together they are called *solid state* detectors. In figure (2.2) is shown the principle in the *scintillation counter*. The *scintillation counter* is mostly applied for research purposes.

2.4 Detectors showing the trace of particles

The *cloud chamber* or the Wilson chamber was designed by Wilson as early as in 1912.

Figur (2.3)



The chamber is equipped with a glass window and a piston and contains water vapour near saturation. The radioactive preparations are placed inside the chamber.
When the piston is pulled out, the vapour in the chamber is cooled, and the vapour becomes in a state of super saturation. The ions, which are formed along the trajectory of a radioactive specimen are, however, sufficient to start a condensation. Along the trajectory will be formed miniscule drops of water which illuminated can be observed or photographed.
If the chamber is placed in a magnetic field, one can determine the momentum of the particles, which have left the trace.
The Wilson chamber has earlier had a great importance in experimental physics, when observing radioactivity or the result of nuclear reactions.
For example it was in a Wilson chamber in 1947, that the first decay of a hitherto unknown particle, (forming a V-formed trace) was observed, when the cloud chamber had been exposed to cosmic radiation.
The resulting trace revealed for the first time a so called "strange particle", the famous Λ_0.
Because of the conservation of "Strangeness", it could not decay via strong interactions, and therefore had a lifetime long enough to leave a trace in the Wilson chamber.

For research purposes the Wilson chamber has been totally ousted by the bubble chamber, the first of which was constructed in 1952.
The bubble chamber contains liquid hydrogen at very high pressure and very low temperature.

The functionality of the bubble chamber is this. If the pressure is shortly diminished, the temperature in the chamber will be above the boiling point of hydrogen.

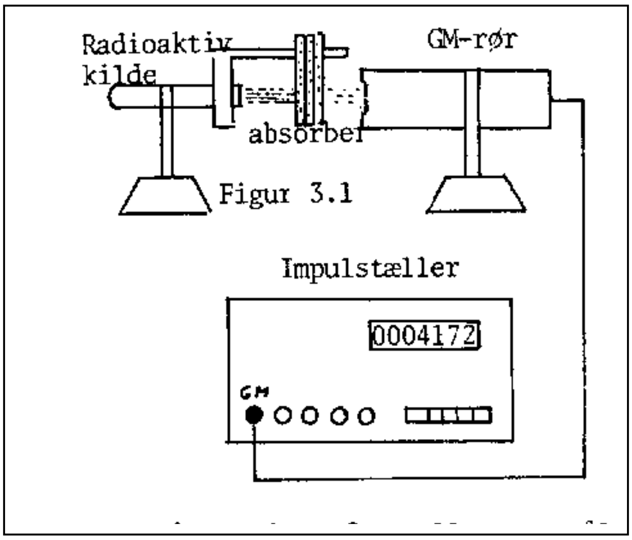
If a charged particle passes the chamber, then tiny bubbles will be formed by ionized hydrogen atoms along the path of the particle and leave a visible trace.
The bubble chamber is placed in a strong permanent magnetic field. The traces of charged particles and the collisions between them are photographed from three orthogonal directions, whereafter computers can calculate the momenta and angles completely.
The bubble chamber has had since the seventies in CERN and elsewhere extensive appliance in the study of collisions between elementary particles. See Ch 10 the section on elementary particles.

3. Absorption of radiation from radioactivity

The alpha-particles are characterized by having a definite range, when passing through various materials. Observation of alpha particles in a Wilson chamber shows a short trace (a few centimetres). α -particles having the same energy, will also have the same range, only submitted to minor fluctuations, the so called straggling.

The explanation is that the α -particles loose their energy by collisions with electrons.
The ionization energy of the atoms is 10 – 100 eV per collisions. Because of the much larger mass (about 5,000 larger than that of the electrons), the α -particles practically exhibit no change of direction when they collide with the electrons.
The mean ionization energy of the atoms in air is 35 eV and therefore an α -particles having energy 3.5 MeV will ionize about 10^5 atoms before it is decelerated.

For the β - and γ -particles the circumstances are somewhat more complicated, since these particles do not have a definite range. Nevertheless there are some simple statistical rules for the absorption of β - and γ -particles in materials.



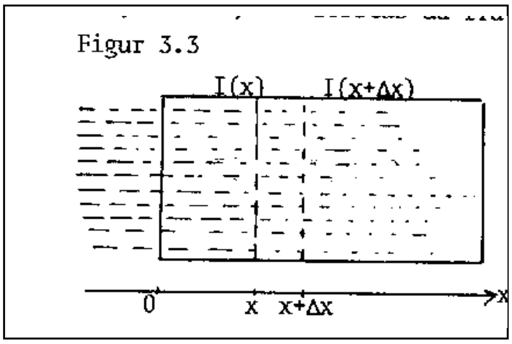
The absorption of the β -particles can be investigated with a line up as shown in the figure (3.1). The beta-source is placed at a fixed position from the detector e.g. a *GM*-tube connected to an electronic counter.
First a count is registered without absorbers.
Then discs of aluminium with various thickness are inserted in between the radioactive source and the detector, starting with 0.50 *mm Al*.
To get a better statistics, the time for counting is adjusted, so that the number of counts in each session are not very different.
For each absorber the intensity is determined in the same manner as *counts/counting-time*.
If the intensity $I(x)$ is mapped as a function of absorber thickness on semi-logarithmic paper (vertical axis logarithmic, x-axis normal) , you will find that the measured intensities approximately lie on a straight line, with a negative slope.
This means that the intensity can be represented by a decreasing exponential function of the absorber thickness.
If a similar experiment is performed with a *gamma*-source, replacing the *Al*-discs with *Pb*-discs, we will find the same exponential decrease, but with a different slope, naturally.

For *beta*- and *gamma* radiation applies the following expression for the dependence of intensity on absorber thickness.

(3.2) $$I(x) = I_0 e^{-\mu x}$$

$I(x)$ denotes the intensity of the radiation after it has passed an absorber thickness of x.
I_0 is the intensity measured without absorber.

μ is called the *linear absorption coefficient*. It depends on the nature of the radiation, the energy of the radiation together with the kind of absorber material. It has the SI-unit m^{-1}.
μ may be calculated theoretically, when the over all scattering cross section is known. (see Ch 9, section 4, for the definition and calculation of cross sections)



That the intensity must be a exponential decreasing function of the absorber thickness can actually be realized from simple reasoning:
The figure (3.3) shows β - or γ -radiation on its way through an absorber.
The intensity at the position x is denoted $I(x)$.
We then argue that the amount of radiation that is absorbed, that is, scattered in the thin layer Δx is proportional both to $I(x)$ and Δx. The absorbed radiation is:

$$I(x) - I(x + \Delta x).$$

From this follows an equation, valid for small Δx.

(3.4) $$I(x) - I(x + \Delta x) = \mu I(x)\Delta x \quad \Leftrightarrow \quad \frac{I(x + \Delta x) - I(x)}{\Delta x} = -\mu I(x)$$

And by letting Δx go to zero.

(3.5) $$\frac{dI(x)}{dx} = -\mu I(x)$$ (Which has the solution)

(3.6) $$I(x) = I_0 e^{-\mu x}$$ (Where $I_0 = I(0)$)

We can see that this simple argument leads to the same dependence that was found experimentally.

By the half-width $x_{½}$, we shall understand the absorber thickness which bisects the intensity.
$x_{½}$ is independent of x (a property of the exponential function), and can be determined by the equation:

$$I(x_{½}) = \tfrac{1}{2} I_0 \quad \Leftrightarrow \quad I_0 e^{-\mu x_{½}} = \tfrac{1}{2} I_0 \quad \Leftrightarrow \quad e^{-\mu x_{½}} = \tfrac{1}{2}$$

Taking *ln* on both sides gives: $x_{½} = \dfrac{\ln 2}{\mu} \quad \Leftrightarrow \quad \mu = \dfrac{\ln 2}{x_{½}}$

If you want to screen for radiation from radioactivity, it is clearly necessary to know the half-width.

3.8 example.
For a certain type of γ - radiation the half width in concrete is found to be 16 *cm*.
Determine the thickness of concrete which reduces the intensity with 99%.
Solution.
We must solve the equation: $I(x_{0.01})$ =0.01I_0. Using the same procedure as above, we get the solution:

$$I(x_{0.01}) = 0.01I_0 \quad \Leftrightarrow \quad I_0 e^{-\mu x_{0.01}} = 0.01 I_0 \quad \Leftrightarrow \quad e^{-\mu x_{0.01}} = 0.01 \quad \Leftrightarrow \quad x_{0.01} = \frac{\ln 100}{\mu}$$

At the same time: $x_{½} = \dfrac{\ln 2}{\mu}$, from which we find: $x_{0.01} = \dfrac{\ln 100}{\ln 2} x_{½} = 6.64 x_{½} = 106\,cm$

4. Definition of units for radiation and doses for activity

The activity of a radioactive specimen is defined as the number of decays per second.
If ΔN is the number of decays in the time Δt, then the activity is defined as:

(4.1) $$A(t) = \frac{\Delta N}{\Delta t} \qquad or \qquad A(t) = \frac{dN}{dt}$$ (Since the activity may vary with time).

The units for measuring the activity have changed (dramatically), since this book was first written, but sometimes the old units are still in use (in older books), so we mention them for completeness.

The traditional unit for activity is Curie:

4.2 **Definition: 1 Curie (1 Ci)** is defined as the activity 3.7 10^{10} disintegrations per second. (The activity of 1 *g* radium).

For experiments in class the highest activity allowed was 5 μCi (1980), but it is probably much lower now (if at all).

The unit Curie has been replaced by the unit Becquerel:

4.3 Definition: 1 Becquerel : Which simply means 1 disintegration per second.

It took about 30 years from the discovery of radioactivity before the departments of health became aware of the danger of radioactivity on humans.
In the early 1900s radioactive preparations were sold as medicine to cure almost anything.
Pierre Curie died early because he carried a specimen of radium in his breast pocket.
Even in the 50ties there were some carelessness concerning X-rays, as we have mentioned earlier.

Today we know that the ionizing effect of radiation can cause biological damage, since it can have a devastating influence on the cells in the human tissue. (Induce cancer).
From a health-physical point of view it is the energy content in the radiation that causes the damage more than the activity.
The ability of ionizing is measured in *Roentgen*, but the unit is no longer in use.

4.4 Definition: 1 Roentgen (1 *R*) is the exposure dose, which in 1 cm^3 of air (at the standard condition) creates a charge of 333 *pC* of each kind (positive and negative).

Since one ion has the charge 1.60 10^{-19} *C* this corresponds to 2.08 ion pairs per cm^3 air, and since 1 cm^3 air has the mass 1.293 10^{-3} *g* , again this corresponds to 1.6 10^{12} ion pairs per gram of air.

By the *absorbed dose* in a body (subject to radiation) is understood the absorbed radiation energy per unit mass. As unit was previously used *rad* (radiation absorbed dose) 1 *rad* = 10^{-2} *J/kg*.
Since the average ionization energy in air is 35 *eV*, it follows that: 1*R* = 35 *eV* 1.6 10^{-19} *J/eV* 1.6 10^{12} g^{-1} = 0.896 10^{-5} *J/g* = 0.896 *rad*.

4.5 Definition:
Nowadays the units *rad* and *Roentgen* have been replaced by the unit *Gray*, which is simply 1 *J/kg*.

The damaging effect of radiation does not only depend on the absorbed dose, but also on biological factors. The earlier name for the dose that takes biological factors into account was the *RBE* dose (Relative Biological Effectiveness). This dose is measured in *rem* (roentgen equivalent man).

RBE-dose = (the biological factor) x (the absorbed dose).

The biological factor is 1 for beta- and *gamma* radiation and about 20 for *alpha* radiation.

4.6 Definition:
Nowadays the *rem* unit is replaced by the unit *Sievert* (*Sv*) . 1 *Sv* = (the biological factor)x 1 *Gray*.

5. Natural radioactivity

Studying radioactivity from earths (natural radioactivity), Rutherford and his collaborators rather rapidly found out that the radiation from radioactive substances came from the atomic nucleus, (discovered by Rutherford only few years earlier).
Since the charge of the nucleus changes by α- or β-decay, then radioactivity is in fact transformations of the elements, (which the chemist, by the discovery of the periodic system of the elements, at last had established as impossible. Something that is still true, when regarding, chemical reactions).

Since Chadwick in 1932 had identified the *neutron* as a component of the nucleus, it has been clear that the nucleus is built from *protons* (which are identical to the nucleus of the hydrogen atom) and *neutrons*, which are neutral particles of the same type as the proton, (but a little heavier).
The mass of the *proton* is 1840 times the mass of the *electron*.

$$m_{proton} = 1.672\,10^{-27}\ kg \qquad m_{neutron} = 1.675\,10^{-27}\ kg$$

The charge of the proton is numerically equal to the charge of the electron: e = 1.602 10^{-19} *C*.

The number of protons in the nucleus is denoted *Z*, which is the same as the atom number in the periodic system of the elements, and also the number of positive elementary charges in the nucleus. The neutron number is *N*. The atomic mass number is $A = Z+N$, equal to the number of protons and neutrons.

A nucleus is then denoted by the chemical symbol of the atom, and with *Z* as index below, and the atomic mass number as index above. If the chemical designation is *X* then of the nucleus is: ${}^{A}_{Z}X$.

For example:

^1_1H : The most common hydrogen nucleus. $Z = 1$, $A = 1$, $N = 0$.

^4_2H : Helium nucleus = α-particle. $Z = 2$, $A = 4$, $N = 2$.

$^{27}_{13}Al$: Aluminium nucleus . $Z = 13$, $A = 27$, $N = 14$.

The same atomic element may have a different number of neutrons in the nucleus. Such atoms which have same number of protons Z , and the same chemical designations are called *isotopes*. Isotopes have the same chemical properties, and can not be separated chemically (or by other atom physical processes), whereas different isotopes may be separated in e.g. a mass spectrograph, because of their difference in mass.

All the elements in the periodic system have isotopes: For example, we may mention: Deuterium (heavy hydrogen): ^2_1H $(Z = 1,\ A = 2, N = 1)$, and the two isotopes of carbon: $^{12}_6C$ and $^{14}_6C$.

In general it is only one of the isotopes that dominate in nature.
From the existing hydrogen in nature 99.985% is ^1_1H , while only 0.015% is deuterium ^2_1H .
For oxygen there exist in nature 3 isotopes: $^{16}_8O$ (99.765%), $^{17}_8O$ (0.0375%), $^{18}_8O$ (0.20%).
This dominance of one isotope means that the atomic masses of the lighter atoms (relative to the mass of hydrogen) are very close to integral numbers.
A flagrant exception is chlorine, which in nature is found in two isotopes: $^{35}_{17}Cl$ (75.4%) and $^{37}_{17}Cl$ (24.6%). The atomic mass for *Cl* can therefore be computed as: 35·0.754+37·0.46 = 35.5.

Since Z is also the number of positive elementary charges in the nucleus, it turns out to be practical, (when counting the total charge), to put $Z = -1$ for the electron.

Protons together with neutrons are called *nucleons*.

The atomic mass number A is thus the number of protons and neutrons in the nucleus.

During nuclear disintegrations or nuclear reactions the following conservation laws apply:

1. Conservation of energy, momentum and angular momentum.
2. Conservation of the collected charge.
3. The number of nucleons is conserved.
4. The number of leptons is conserved. (The electron and the neutrino are leptons)

Using these conservations laws it is possible to express the radioactive disintegrations in symbolic equations. (Not to confuse with chemical reactions). The conservation of Z and A, are secured, since the numbers in the upper index and the lower index should add up to the same number on both sides of the reaction schema.

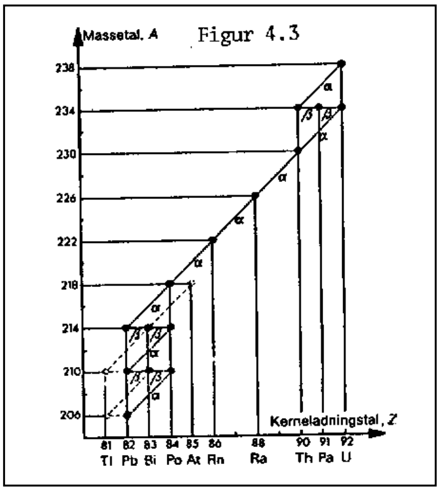
$^{238}_{92}U \to ^4_2He + ^{234}_{90}Th$ (Emission of an α-particle from a uranium nucleus)

${}^{234}_{90}Th$ itself is also radioactive and emits a β-particle and is converted to *Pa* (Protactinium)

$${}^{234}_{90}Th \rightarrow {}^{0}_{-1}e + {}^{234}_{91}Pa$$

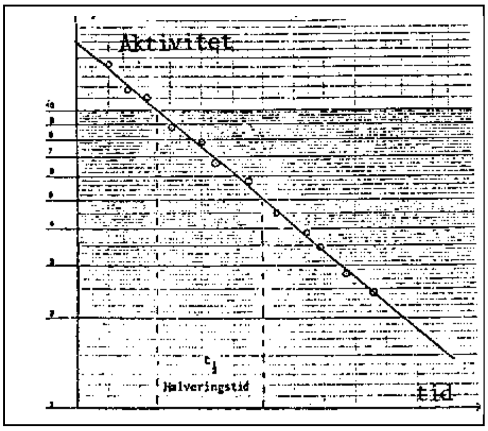
${}^{234}_{91}Pa$ is also radioactive, since it emits a β-particle, and is converted to an isotope of uranium.

$${}^{234}_{91}Pa \rightarrow {}^{0}_{-1}e + {}^{234}_{92}U$$



${}^{234}_{92}U$ is also radioactive and emits an α-particle. From here follows a long series of decays, ending with a stable isotope of *Pb.* The first three decays in the disintegration chain are specified above, but the rest of the decays follow the same conservation rules. This series is called the *uranium-series*.
The whole series is shown in figure (4.3) to the left.
The nuclei in the series have very different half lives. The half-lives are shown in the table (4.4) on the next page.
Besides the uranium series there are two other series starting with ${}^{235}_{92}U$ and ${}^{232}_{90}Th$, and they are called the actinium and the thorium series.
In laboratories there have been produced (artificial) radioactive specimens of nearly all the elements. These radioactive samples are often formed by bombarding nuclei with neutrons.

6. The exponential law of decay. Half-live of radioactive nuclei



If you measure the activity of a radioactive sample, that is, the number of decays per second, and plot the activity in semi-logarithmic paper, you will in all cases find that the points lie on a straight line with a negative slope. This means, as we know, that the activity $A(t)$ is a decreasing exponential function of time.

(6.2) $$A(t) = A_0 e^{-kt}$$

The constant k is called the decay-constant or the disintegration-constant. It has the *SI*-unit s^{-1}.
k does not depend on the amount of the sample, but only of the nature of the radioactive sample.

Table (4.4) Half lives of radioactive nuclei	
^{238}U	4.5 10^9 *y*
^{234}Th	24 *days*
^{234}Pa	1.2 *min*
^{234}U	2.5 10^5 *y*
^{230}Th	8.0 10^4 *y*
^{226}Ra	1620 *y*
^{222}Rn	3.8 *days*
^{218}Po	3.1 *min*
^{214}Pb	26.8 *min*
^{214}Bi	19.7 *min*
^{214}Po	0.16 *ms*
^{210}Pb	22 *y*
^{210}Bi	5.0 *days*
^{210}Po	138 *days*

From (6.2) we can see that $A(0) = A_0$ the initial activity.
The time which passes, until the activity is halved is called the half-life of the radioactive substance.
The half life does not depend on *t*. We denote the half-life $t_{½}$.
The half-life can be expressed by the decay-constant, using (6.2).

$$A(t_{½}) = \tfrac{1}{2} A_0 \quad \Leftrightarrow \quad A_0 e^{-kt_{½}} = \tfrac{1}{2} A_0 \quad \Leftrightarrow \quad e^{-kt_{½}} = \tfrac{1}{2}$$

(6.3) $$t_{½} = \frac{\ln 2}{k}$$

The last expression is obtained by taking the logarithm on both sides. If the activity is mapped as a function of time, the half-life can be found by measuring the distance between to points $t_{½} = t_1 - t_2$, where $A(t_2) = ½\, A(t_1)$.
The decay-constant can be found by solving (6.2) with respect to *k*.
The half- lives for radioactive nuclei differ a lot, from 4.5 10^9 years to 0.16 *ms*. For example it is seen that *U-238* has a half-life compared to the age of the universe. It is exactly on the grounds of the ratios between the amounts of the different nuclei in the decay-series, it has been able to get an overview over when the uranium-238 was created, and by then an assessment of the age of the universe.

We shall now give a statistical explanation of the law of decay. This law is actually one of the most conspicuous examples of *the indeterminism in quantum physics*.

It turns namely out that in principle it is impossible to predict when a nucleus will decay, even if you know precisely when it was formed. The “life of a nucleus” is submitted to statistical fluctuations, something which is totally unheard of in classical mechanics.

We shall therefore assume that a nucleus has the same probability *k* to decay every unit of time from the moment it was created, independently of how long it has “lived” (Rather much different from the course of the lifetime of a human being).

The probability that the nucleus will decay in the (small) time Δt is then $k\Delta t$.
Let $P(t)$ denote the probability that a nucleus has not decayed in the interval [0, t]. $P(0) = 1$.
Using the conditions above, it is possible to make a differential equation for the probability $P(t)$.

$P(t + \Delta t)$ is the probability, that a nucleus that has not decayed at *t* , does not decay in Δt , the latter has the probability $1 - k\,\Delta t$, so:

(6.5) $$P(t+\Delta t) = P(t)(1-k\Delta t) \quad \Leftrightarrow \quad \frac{P(t+\Delta t)-P(t)}{\Delta t} = -kP(t)$$

By letting Δt go to zero we then find:

(6.6) $$\frac{dP}{dt} = -kP(t)$$ which has the solution

$$P(t) = e^{-kt}$$ since $P(0) = 1$

The probability of "survival" of a nucleus decreases accordingly exponentially with time.

If we at time t = 0 have $N_0 = N(0)$ identical nuclei, we should statistically expect that we are left with $N(t) = N_0P(t)$ nuclei at time t. This is precisely what is expressed in the law of decay.

(6.7) $$N(t) = N_0 e^{-kt}$$

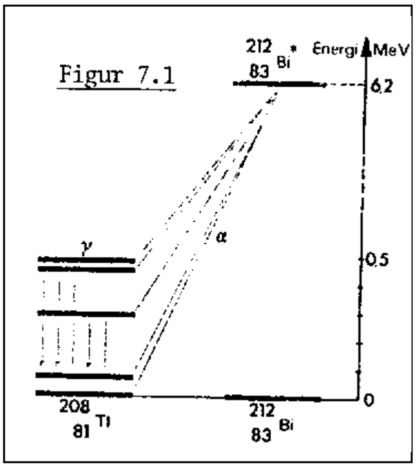
The activity $A(t) = -\dfrac{\Delta N}{\Delta t}$ is defined by the number of decays – ΔN the time interval Δt (the minus sign because ΔN is negative), and letting Δt go to zero, we find:

(6.8) $$A(t) = -\frac{dN}{dt} \quad \Leftrightarrow \quad A(t) = kN_0 e^{-kt} \quad \Leftrightarrow \quad A(t) = A_0 N_0 e^{-kt}$$

From (6.8) we also see that we may always find the activity as kN. $A(t) = kN(t)$.

7. Energy issues of radioactivity

Radioactivity means the emission of a particle from the atomic nucleus.
As it is the case for the atom, the structure of the nucleus is characterized by a series of separated *stationary energy levels*.



Before and after the emission of a α-, β- or γ-particle is the nucleus in a *stationary state*, and you should therefore expect that the energies, with which the α- β- γ particles are emitted should have the character of a line spectrum, with single well defined energies, (as it is the case of light emitted from atoms).

It turns out that this is precisely the case for the α- and γ-radiation. The variation of the energies for the α-particle can be explained since the new nucleus after having emitted the α-particle does not necessarily returns to its ground state, but rather to an exited state of the "daughter-nucleus".
The exited nucleus will hereafter return to its ground state, by emitting one or more γ-particles.

This situation is illustrated in the figure (7.1), using ${}^{212}_{83}Bi$ as an example. The α-reaction is: ${}^{212}_{83}Bi \rightarrow {}^{4}_{2}He + {}^{208}_{81}Tl$. The daughter-nucleus ${}^{208}_{81}Tl$ may be in 4 exited states besides the ground state. In the first case the transition to the ground state will happen by emitting one or more γ-particles.

By doing precision measurements of the energy of the emitted α-particle and the accompanying γ-particles, it has been possible to determine the energy levels of many daughter-nuclei from decays of a radioactive mother nucleus.

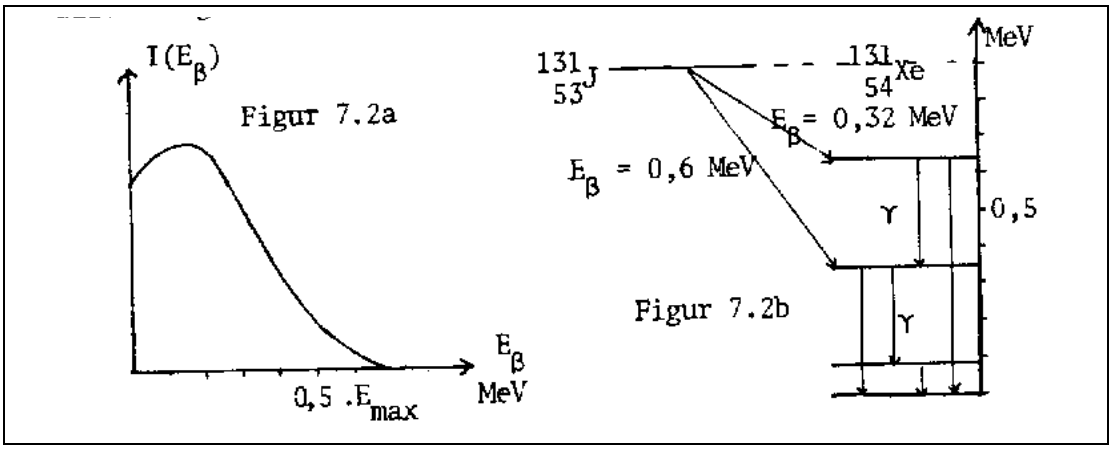
The energy levels of the atomic nuclei have many similarities with that of the atom. The crucial difference is the size of the energies and energy levels. While the atom is kept together by long range electrical forces (Coulomb's law), the nucleus is held together by the *strong interactions*. This is reflected in the energies of the emitted photons (gamma-radiation).

While the visible photons from the atom have energies from 1*eV* to 1000 *eV*, then photons emitted from the nuclei (γ-particles) have energies in the interval from 10^3 *eV* to 10^7 *eV*.
We remind you that γ-radiation as well as light is electromagnetic radiation.

7.1 The discovery of the neutrino

When emitting a β-particle it is basically the same that happen as when emitting a α-particle, namely a transitions between two energy levels in the mother and daughter nuclei.

It is therefore somewhat surprising that the β-spectrum is a continuous spectrum.
Experimental measuring of the energies of β-radiation demonstrates that for the same β-active nucleus, the emitted electrons can have energies from 0 to a certain maximum energy E_{max}.



The figure (7.2a) shows a typical β-spectrum. On the 1. axis is allocated the energy E_β, and on the 2. axis is allocated the relative intensity: $I(E) = |dN|/dt$ such that $|dN| = I(E)dt$ is the area of the strip having the width *dE*, denotes the relative number of particles emitted with energy in the interval [*E, E*+*dE*].
In figure (7.2b) is shown the energy levels and the released energies for the β-decay of ${}^{131}_{53}I$ into ${}^{131}_{54}Xe$. It turns out that the observed maximum energy of the electrons exactly match the difference in the energy levels. Since the energy E_{max} has been released, it became a mystery where the rest of the energy E_β - E_{max} had gone.

Very accurate experiments performed in 1931 revealed that no other detectable particle was emitted along with the β-decay. So the results of the experiments were apparently a break with the energy conservation theorem which, on the other hand, was totally unacceptable from a theoretical point of view.
However, the German physicist W. Pauli postulated in 1932 that the electron always was emitted together with a neutral mass-less particle that carried away the excess of energy in the β-decay. Since the particle could not be detected, it could have no electrical properties (electromagnetic interactions). The hypothetical particle was given the name *neutrino* (υ).

It is in itself surprising that the positive charged nucleus can emit negative charged electrons, but Pauli imagined that the β-decay happened so that a *neutron* was converted to a *proton* by emitting an electron and a neutrino. In the particle physics, one has chosen to call the particle emitted together with the electron for an anti-neutrino ($\bar{\nu}$). The electron and the neutrino are both so called leptons. The electron is a lepton, and a conservation rule says that the lepton number must be conserved, so the neutrino must be an anti-lepton. The β-decay of a nucleus can then be written:

$$(7.3) \qquad {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \overline{\nu} \quad or \quad {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \overline{\nu}$$

Free neutrons are unstable and decay to proton-neutron-antineutrino having a life-time 16.7 *min*.

The existence of the neutrino was still hypothetical until the Italian physicist Enrico Fermi in 1934 brought forward a theory for the β-decay.
He proposed that besides the already known interacting forces of nature (gravitation, electromagnetic, and nuclear forces) there existed another form of interacting force, which he called *weak interactions*. All elementary particles have weak interactions, but the neutrino has *only* this extremely weak interaction.

Based on this theory Fermi was able to predict the continuous spectrum of the β-decay.
The very fine agreement between theory and experiment confirmed the belief in the existence of the neutrino, although it had yet not been detected.

It should take 25 years before a group of American physicists succeeded in establishing the existence of the neutrino experimentally.

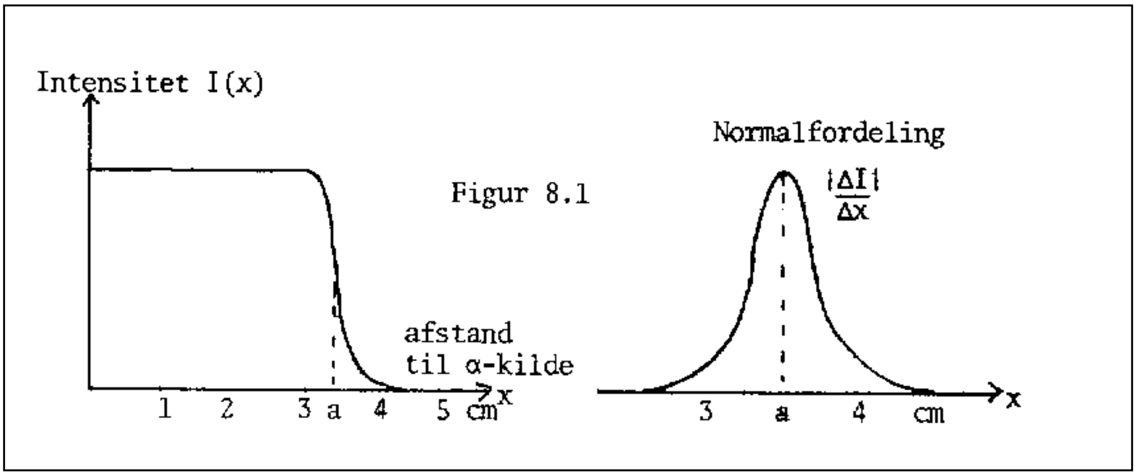
The challenges, however, seemed enormous. From nuclear reactions in the sun, the earth is hit by 10^{12} neutrinos per second. On the average only one of these neutrinos will trigger a nuclear reaction, the others will pass through the earth without interactions of any kind.
In the American experiment they succeeded to register a few neutrino events in some gigantic tanks Buried deep in the ground shielding them especially from γ-rays, and monitored by a gigantic number of detectors.
From the mid sixties it has been possible to detect many neutrino reactions, since when emitted from an accelerated particle generated from the monstrous accelerators the neutrinos can obtain very large energies and the probability of a neutrino interaction grows with the square of their energy.

8. The radioactive radiations interaction with matter

We have earlier mentioned that the α-particles have a well defined range, and that the intensity from the β– and γ-decay decreases exponentially with the thickness of the absorber. Here we shall discuss the processes which are the causes why the radioactive particles are decelerated or scattered.

The deceleration of the α-particles is (almost only) due to collisions with the electrons in the atoms. The α-particles collide with the electrons in the atoms, which therefore become ionized.
Since the mass of the α-particle is about 7000 times the mass of the electron, there will be no change of direction of motion for the α-particle after such a collision.

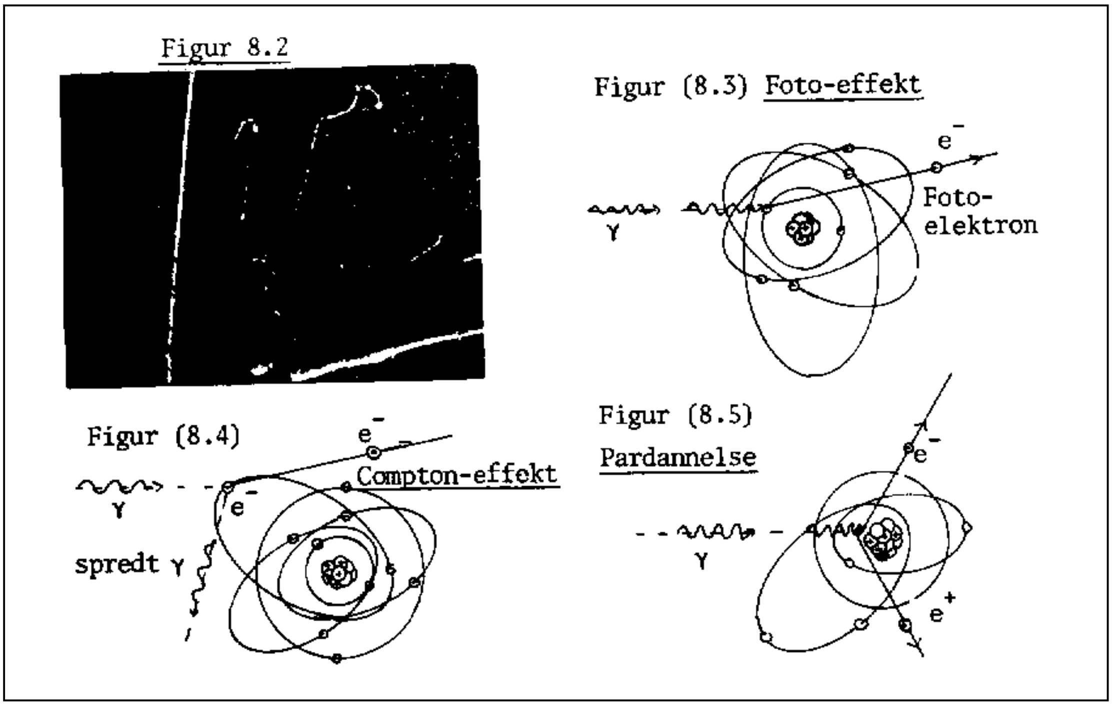
The α-particles are therefore seen as a short thick trace in a Wilson chamber or in a bubble chamber. Once in a while a crack on the trace is seen, which means that the α-particle has hit a nucleus.
The mean ionization energy for atoms in air is 35 *eV*, and an α-particle having the energy 3.5 *MeV* must therefore ionize 10^5 atoms before it is stopped.



In figure (8.1) to the left is schematically shown the result of a measurement of range for α-particles in air.
The experiment is simply carried out by placing a (scintillation) counter at various distances from the α-particles source and measuring the intensity. If the intensity is mapped versus the distance x, you will get a graph shown to the left. The intensity is practically constant until it suddenly drops to zero.
The mean range is read as the median. Investigating a little more closely, how the range varies, one gets a result shown to the right. The graph shows that the range is normally distributed with the mean equal to the median.

The decelerations of the β-particles are almost exclusively cased by collisions with the electrons. That the β-particles do not have a definite range, have several causes.
Firstly, as we have mentioned earlier, the β-particles are not mono-energetic, since the neutrino carries away a varied part of the energy released in the decay.
Secondly the β-particle, which is an electron, looses a varied part of its energy in a collision with an electron.
The number of collisions that a β-particle has had before it is decelerated is therefore subject to much larger fluctuations than it is the case with the α-particles.
Thirdly, the β-particles may loose their energy by braking-radiation (X-rays) . The braking radiation dominates the energy loss by medium energies of the β-particles, while ionization is more significant at lower energies.

Finally the directional change in a electron-electron collision vastly larger than the collision of an α-particle with an electron. After a β-particle has had a collision with an electron it will in most cases disappear from the beam, and will no longer contribute to the intensity
In the figure (8.2) is shown a cloud-chamber exposure of β-particles as well as α-particles.
The short heavy traces come from the α-particles.



The absorption of γ-radiation happens in various processes. At lower energies the photoelectric effect dominates. By the photoelectric effect the γ-photon looses all of its energy, by throwing out an electron from the atom.
The emitted electrons are called photo-electrons, and they are emitted with an energy given by the photoelectric law of Einstein: $h\nu = \frac{1}{2}mv^2 + E_0$.
E_0 is the binding energy of the electron in the atom, and υ denotes as usual the frequency of the photon. In figure (8.3) is schematically illustrated the photoelectric effect.
(The photoelectric effect is thoroughly discussed in chapter 6).

At higher energies the absorption of γ-rays is dominated by the Compton-effect.
The Compton-effect may be perceived as a collision between a γ-particle and a loosely bound electron. By the collision the γ-particle looses some of its energy, and gets a change of direction.
In figure (8.3) is schematically illustrated the Compton-effect.

At even higher energies (E_γ > 1 MeV) occurs a new phenomenon called *pair-creation*.
When a γ-particle collides with a heavy nucleus, its energy is converted in the creation of an electron–positron pair. (The positron is the anti-particle to the electron. It has exactly the same mass as the electron, but has the charge +*e*, in contrast to the electrons charge –*e*) .

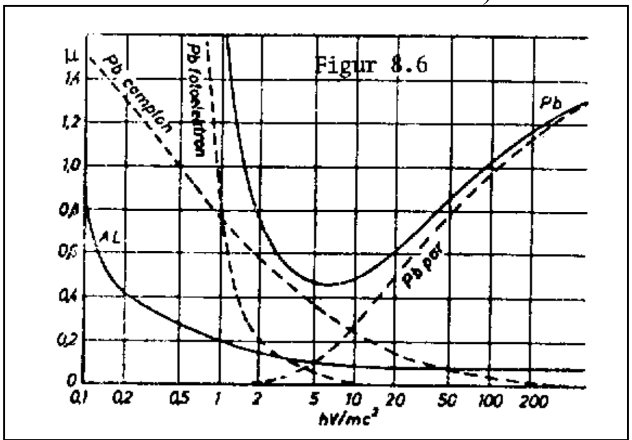
Pair creation is one of the most direct confirmations of Einstein's equivalence between mass and energy. If we convert the mass of the electron: m_e = 9.11 10^{-31} *kg* to its equivalent energy we find:

$$\begin{aligned} E &= m_e c^2 = 9.11 \cdot 10^{-31}\ kg (3.00 \cdot 10^8\ m/s)^2 = 8.20\,10^{-14}\ J \\ E &= 8.20\,10^{-14} / 1.602\,10^{-19}\ eV = 0.511\,MeV \end{aligned} \tag{8.5}$$

Since there always are created to particles each having a rest energy 0.511 *MeV*, the γ-particle must have an energy at least 1.022 *MeV* to make a pair creation possible.

If the energy of the γ-particle is larger than the threshold for pair creation the surplus energy is transformed to kinetic energy of the electron-positron pair, (since the γ-particle always disappear completely after the pair-creation has taken place).
Pair creation always takes place in the vicinity of a heavy nucleus, which can absorb the excess of momentum from the γ-particle. This is easy to understand if the γ-particle has the energy close to 1.022 *MeV*, so that the electron–positron pair are at rest.
But also if the γ-particle has an energy that is above the threshold for pair creation, one may from the conservation of energy and momentum show that it is necessary to have a heavy particle to absorb the momentum of the γ-particle.

After decelerating, the positron may be captured by an electron, resulting in the creation of two γ-0.511 *MeV* particles in a so called *annihilation*. (Creation of a single γ-particle is excluded for the reason of conservation of momentum).



In the figure (8.6) is shown how the absorptions coefficients for the photo-electric effect, the Compton-effect and pair creation depend on the energy of the γ-particle.

The linear coefficient of absorption μ is found by adding the coefficients for photo-effect, Compton scattering and pair-creation.

$$\mu = \mu_{photo} + \mu_{Compton} + \mu_{pair\ creation}$$