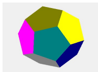
Driving the wall of death
and
related mechanical problems



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1. How can you drive round on a vertical wall without falling down?

When I was a boy (some 60 years ago), there were these travelling amusement parks where, among other entertainments, was presented driving the “wall of death”.
For a twelve year old boy this kind of entertainment, had absolutely a significant attraction.
Paying for the ticket, I entered a large tent, where was built a large vertical wood cylinder, reinforced on the outside with wooden beams.

The show began with one man on a motorbike drove around in the bottom gaining speed, and then speeding up an oblique ramp, and finally with great speed driving in a horizontal circle.

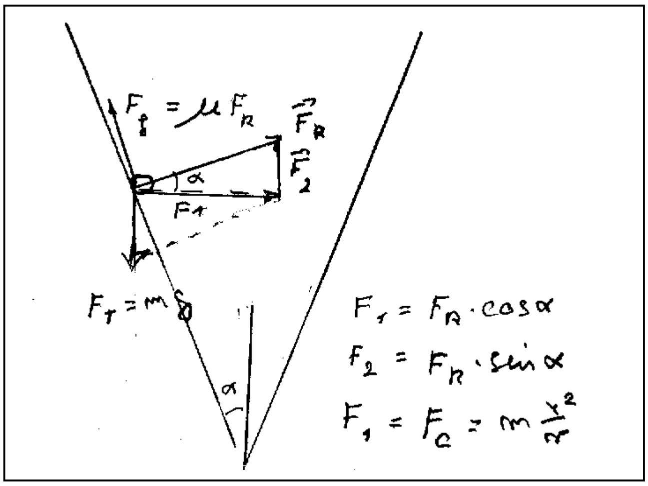
The motorbike exhausted plenty of smoke, and the wooden cylinder crashed with loud noises, each time the motorcycle passed.
The driving was repeated with another stunt man (without helmets), who drove without hands on the steering wheel, and he also drove lying flat on the bike.
Then the two motorcycles raced together taking each other over.
Finally a small racing car drove on the vertical wall. Naturally the enthusiasm had no end.

Although it appeared strange, that the vehicles did not fall from the vertical wall, I did not think so much in Newtonian mechanics at that time.
The explanation that the vehicles pressed so hard against the wall, prevented them in falling down, was in fact good enough for the time, but as a physicist it is less satisfactory than finding the relation between the radius of the rotunda, the speed of the vehicles, and possibly a frictional coefficient between the walls and the rubber wheels.

I think that driving the “wall of death“ was prohibited in Denmark in 2011, but on You Tube, you may still find a video recording made in Christiania in 2010.

Before we shall analyze driving in “the wall death”, we shall look into another example, which is mechanical equivalent, but more direct to deal with.

1.1 A bullet which performs in a horizontal motion in a cone



We shall consider a spherical bullet, which performs a circular rolling motion in a cone. Half the top angle of the cone is designated α. First we shall assume that there are no friction between the bullet and the walls of the cone, and that the bullet performs a pure rolling.
In this case the bullet is only affected by gravity $F_T = mg$ and the reaction force from the inner surface of the cone F_R.
The resultant of the two forces F_T and F_R is seen to be the horizontal component F_1 .
From the figure we can see:

$$F_1 = F_2 /\tan \alpha = mg/ \tan \alpha$$

When the bullet performs a uniform circular motion, the resulting force is equal to the centripetal force. So.

$$F_1 = F_c \quad \Leftrightarrow \quad \frac{mg}{\tan\alpha} = m\frac{v^2}{r} \quad \Rightarrow \quad v^2 = \frac{rg}{\tan\alpha} \tag{1.1}$$

If we put $r = 0.5\ m$, and $\alpha = 30^0$ (corresponding to a height $h = r/\tan 30 = 0.87$ m), we find:

$$v = \sqrt{\frac{rg}{\tan\alpha}} = 2.9\ m/s\ .$$

We can, however, see that when α goes to zero the velocity needed goes to infinity, and from this we conclude that a circular motion on a vertical circular wall is not possible, which really is not surprising.

To make it possible, we must add a frictional force between the bullet and the wall. This force has only a component along the wall of the cone, since the motion of the bullet vas considered a pure rolling.

The frictional force is: $F_f = \mu F_N$, where μ is the frictional coefficient and F_N is the normal force, that is, perpendicular to the surface. In this case it is equal to the reaction force F_R.

$$F_N = F_R = \frac{F_c}{\cos\alpha} = \frac{mv^2}{r\cos\alpha} \quad \text{and thus:} \quad F_f = \mu F_R = \frac{\mu F_c}{\cos\alpha} = \frac{\mu m v^2}{r\cos\alpha}$$

For the vertical force on the bullet to be zero, we must therefore have:

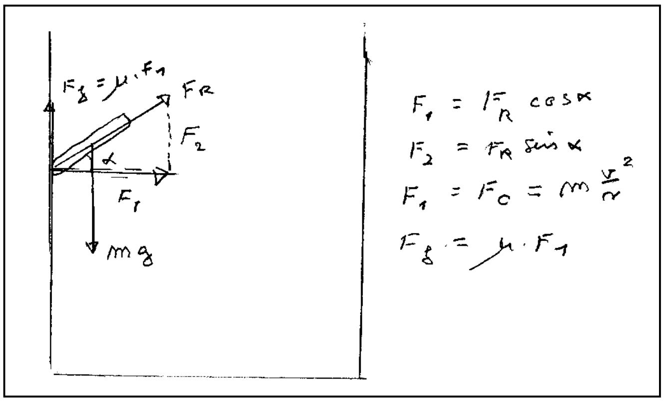
$$F_R \sin\alpha + \mu F_R \cos\alpha = mg \qquad F_R(\sin\alpha + \mu\cos\alpha) = mg$$

$$\frac{mv^2}{r\cos\alpha}(\sin\alpha + \mu\cos\alpha) = mg \quad \Leftrightarrow \quad v^2(\tan\alpha + \mu) = rg \quad \Leftrightarrow$$

$$v^2 = \frac{rg}{\tan\alpha + \mu} \tag{1.2}$$

From (1.2) we can see that including friction the bullet may uphold a circular motion and with a lower speed. And also that it is theoretical possible to do it on a vertical cylinder wall. In that case the velocity is given by the equation: $v^2 = \frac{rg}{\mu}$.

1.2 The wall of death. Motion in a vertical cylinder



We shall then look at the situation, where a motorbike moves on the wall in a vertical cylinder, that is, driving the wall of death. First we shall assume that there is no friction between the wheels and the wall The motorbike (only one wheel is drawn in the figure) is affected by gravity F_T and the reaction force F_R from the wall of the cylinder. The wheels are assumed to have an angle α with horizontal.

Without friction the resulting force is equal to the vector sum of the reaction F_R and gravity F_T.
If the motorbike performs a uniform circular motion, the resulting force must be equal to the centripetal force $F_c = \frac{mv^2}{r}$

From the figure we can see that: $F_c = F_1 = F_R \cos\alpha$.
If the sum of the vertical forces should be zero, we must have.

$$F_2 = mg \quad \Leftrightarrow \quad F_R \sin\alpha = mg$$

If we insert: $$F_R = \frac{F_c}{\cos\alpha} = \frac{mv^2}{r\cos\alpha},$$

We find (again):

(1.3) $$\frac{mv^2}{r\cos\alpha}\sin\alpha = mg \quad \Rightarrow \quad v^2 = \frac{rg}{\tan\alpha}$$

This is the same equation as with the bullet in the cone, and again we see that a circular motion with a horizontal motorbike is not possible.
Assuming that in a cylinder with a radius 5.0 *m* the wheel has an angle α = 10^0 with horizontal, we may however calculate the speed necessary for the motion:

$$v = \sqrt{\frac{rg}{\tan\alpha}} = 16.8\, m/s = 60.6\, km/h$$

Which is far a too high speed, so we must include friction

If you look at video recordings of motion in a death wall cylinder, the bikes actually move horizontally, and as well as the racer cars.

Including friction the resulting force is now the vector sum of gravity $F_T = mg$, the reaction force F_R and the frictional force $F_f = \mu F_N$ where μ is the frictional coefficient between the wheels and the wall, and F_N is the normal force, which in this case is equal to the centripetal force F_c.

We know that when a body performs a uniform circular motion, then the resulting force is directed towards the centre of the motion, and is equal to the centripetal force. $F_c = \frac{mv^2}{r}$.
From the figure, we can see that $F_c = F_1 = F_R \cos \alpha$. The normal force is $F_N = F_c$, so the frictional force is $F_f = \mu F_c$. The sum of forces in the vertical direction should be 0.

(1.4)
$$F_2 + F_f = mg \quad \Leftrightarrow \quad F_R \sin\alpha + \mu F_c = mg \quad \Leftrightarrow \quad \frac{mv^2}{r\cos\alpha}\sin\alpha + \frac{\mu m v^2}{r} = mg$$
$$\frac{v^2 \tan\alpha}{r} + \frac{\mu v^2}{r} = g \quad \Rightarrow \quad v^2 = \frac{rg}{\tan\alpha + \mu}$$

We notice that it is the same condition that we obtained with the bullet rolling in the cone.

Assuming that $\alpha = 0$, $r = 5.0\ m$ and $\mu = 1$, we find for the velocity: $v = \sqrt{rg} = 7.0\ m/s$

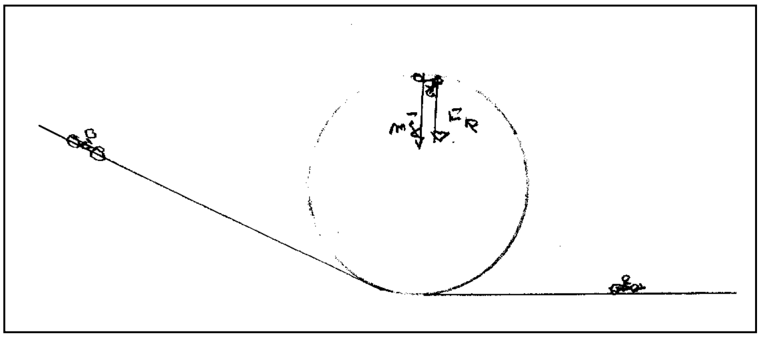
The calculated speed corresponds, however, to the least speed to uphold the motion. An increased speed will only increase the reaction force and thereby the frictional force.

Although the sum of the vertical forces equals zero, gravity will still have a torque $\vec{H} = \vec{r}_G \times m\vec{g}$ acting at the point of contact, where $\vec{r}_G$ is the vector from this point to the centre of gravity of the vehicle.
This torque will tend to turn the vehicle along a horizontal axis, and in a static situation would make it fall down.
The explanation why it does not happen is, however, the same as for a top spinning along a non vertical axis.
When the vehicle rotates in the cylinder it has a large vertical angular momentum along the axis of rotation $\vec{L}_0 = \vec{r} \times m\vec{v}$, where r is the radius of the circular motion.
Since $\vec{H} = \frac{d\vec{L}}{dt}$, then in the time Δt the torque from gravity will make a small horizontal contribution $\Delta\vec{L} = \vec{r}_G \times m\vec{g}\Delta t$, so that the total angular momentum is $\vec{L}_0 + \Delta\vec{L}$.
But this will not cause a fall to the ground, but rather a precession of the axis of rotation, (as it is the case with a top) .
The angular velocity may as it is the case of a spinning top be calculated as: $\Omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta L/\Delta t}{L_0}$.
Assuming the values $r = 5.0\ m$, m = 200 kg, v = 10 m/s and $r_G = 1.0\ m$. we find the angular velocity in the precession of the rotation axis to be to be 0.2 *rad/s*.
This must of course be corrected by the driver.

1.3. Looping the loop

In most amusement parks there are roller coasters, where a carriage performs a vertical circular motion. Why does the carriage not fall down in the upper position? Something, which is not surprising, once you have tried to sling a mass bound to a cord in a vertical circle.

And it certainly does, if the velocity in the upper position is not big enough.



In the upper position the carriage is affected by gravity $F_T = mg$ and the reaction force F_R from the rail. They both have the same direction downwards, and together they form the resulting force on the carriage. When a body perform a circular motion the resulting force is equal to the centripetal force $F_c = F_T + F_R$.

$$F_c = \frac{mv^2}{R}$$

The condition that the carriage remains on the rail is then: $F_R > 0$:

(1.5) $$F_R = F_c - F_T > 0 \quad \Leftrightarrow \quad \frac{mv^2}{R} - mg > 0 \quad \Leftrightarrow \quad v^2 > Rg$$

For a wheel with radius $R = 5\ m$, it results in the modest speed $v = \sqrt{Rg} = 7.0\ m/s = 25\ km/h$

For a wheel with radius $R = 10\ m$, it gives: $v = \sqrt{Rg} = 10\ m/s = 36\ km/h$.

Recall however that these are the minimum speeds, and where friction is not taken into account. In practice the speeds are probably somewhat higher.

Neglecting friction, we would like to calculate from what height the carriage must be released to perform the loop. The energy in the top position is:

(1.6) $$E = E_{kin} + E_{pot} = \tfrac{1}{2}mv^2 + mg2R\ .$$

In the limiting case $F_R = 0$ we have however:

$$F_{res} = F_c = F_T = \frac{mv^2}{R} = mg \quad \Rightarrow \tfrac{1}{2}mv^2 = \tfrac{1}{2}mgR$$

The energy is then: $E = E_{kin} + E_{pot} = \tfrac{1}{2}mv^2 + mg2R = \tfrac{1}{2}mgR + mg2R = \tfrac{5}{2}mgR$

The sufficient energy in the limiting case is therefore: $\tfrac{5}{2}mgR$, so the carriage must be released at least at a height $\tfrac{5}{2}R$.

A more detailed treatment of the problem is given in:

Elementary physics 2, motion in the plane, uniform circular motion.