Astrophysics

Chapter 11 of the textbook Elementary Physics 3

This is an article from my home page: www.olewitthansen.dk

Ole Witt-Hansen

1980 (2017)

Informative remark: The present chapter is a translation from the Danish textbook: Elementary Physics 3. However, the texts, when it appears in the figures are not translated. On the other hand the figures and the supplementing text should speak for themselves.

Contents

1. Introduction

The astronomy is rightly considered as the oldest of all sciences. Astronomic observations have taken place in several thousand years before Christ. Until 1900 the astronomers have essentially been occupied in studying the motion of the planets among the fixed stars.
From Newton's laws of mechanics and his law of gravitation (about 1700), we have gained a complete understanding of the motion of the planets and other celestial objects mutual motions. The practical calculations in predicting the motion of the planets have, however, first been overcome with the invention of the large digital computers after 1960.

The traditional astronomy has in the last 60 years been less important, and the astronomical observations have been more focused on the *galactic astronomy* of the universe.
Not until 1924 it has been clear that the Milky Way's billions of stars is an enormous rotating system of stars, which is called a *galaxy*.
The sun is only one of about 200 billions of stars, which belong to this galaxy. As it appears, when you look in the sky at night, the galaxy is flattened, rather discos formed in the central parts, where the density of stars is highest.

The diameter in our galaxy is about 100,000 light years (*ly)*, and the sun is placed at a distance of 30.000 *ly* from the centre of the galaxy. The sun moves around the centre of the galaxy with a speed of 250 *m/s*.
(1 light year is the distance that the light travels in one year. When converted to meters:
1 light year = $3.0\ 10^8$ *m/s* 365·24·3600 *s* = $9.5\ 10^{15}$ *m*).

With the construction of very large telescopes from the 1920th , it has been discovered that objects which were previously considered to be distant stars in reality were distant galaxies. On the next page is a picture of one of the most spectacular galaxies: The Andromeda galaxy, which was identified already in 1924. The distance to the galaxy is 2.2 million light years. Since then there have been identified millions of other "stars" as very distant galaxies.

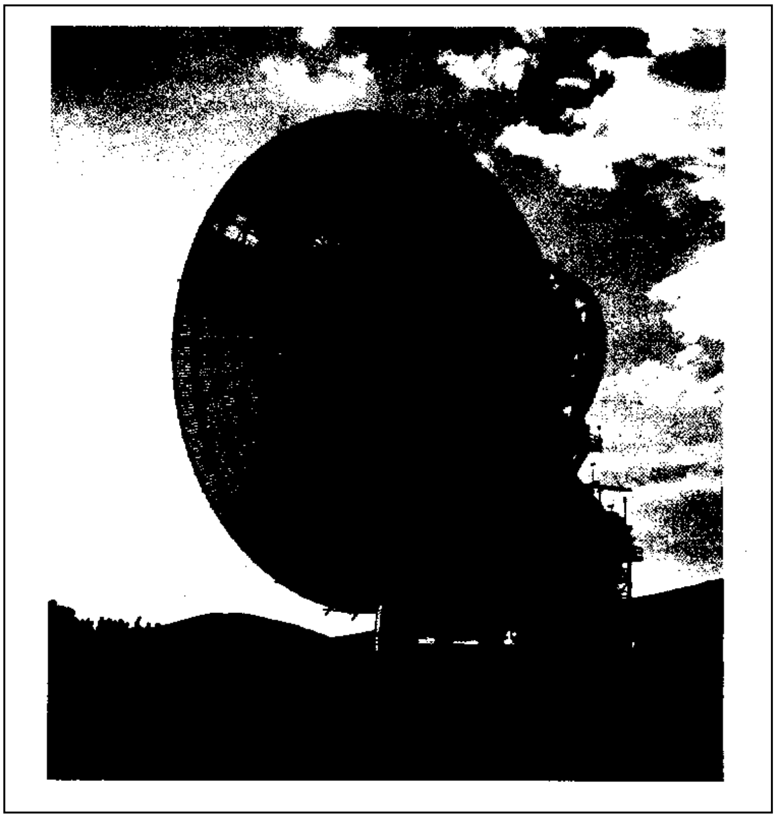
As a consequence of the development of the galactic astronomy a new branch of the astronomy has emerged, which is called *cosmology*.
The cosmology occupies itself with the structure of the universe, its creation and development on a large scale. Since the twentieth all cosmological theories are founded on Einstein's general theory of relativity from 1916. (www.olewitthansen.dk General relativity and Cosmology).

Astrophysics is concerned with the physical processes, which cause the formation of stars and their radiation and their source of energy. In astrophysics the description of formation and development of the stars come from physical models, based on atomic and nuclear physics.

The main part of the electromagnetic and other radiation, which enters the earths atmosphere is absorbed. The atmosphere is, however, almost transparent against visible light. This is called the "optical window" to the universe. The main part of the information about the stars still comes from the optical window.

From the forties it has been recognized that radio-waves in the *cm* band can penetrate the atmosphere without being substantially absorbed. This wavelength region is called the radio window.

The optical window has served to perform astronomical observations for over 5000 years, while the radio-window has only been seriously in use since the 1960-ties.
The building of large radio-telescopes to receive and analyze the radio radiation from space has added the radio astronomy as an important tool in the service of astrophysics. See figure below.

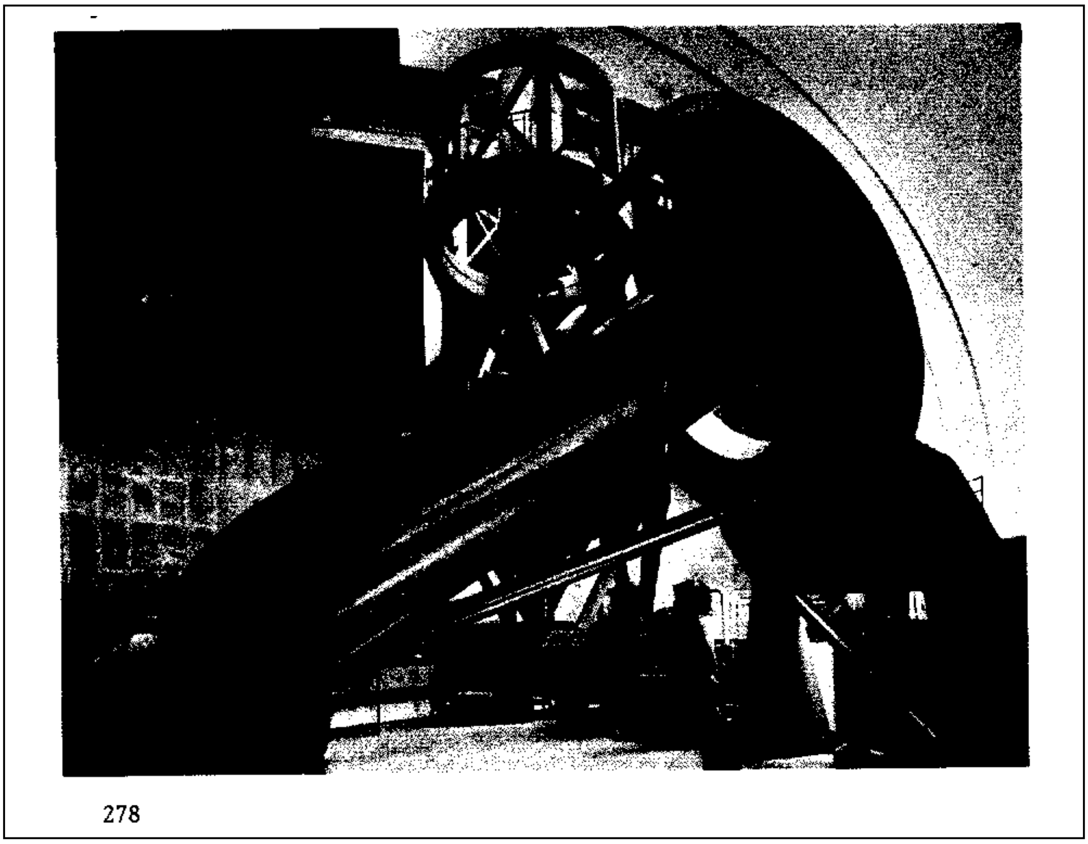


The photo below shows one of the radio telescopes, which is applied both for radio astronomy and communication with space probes. The telescope is placed in a dessert in California.
It has a diameter of 65 meters, and it can be adjusted so that the direction is fixed to a celestial object, irrespectively of the rotation of the earth.

The radio telescopes have especially shown their great importance by their ability to receive signals from objects (quasars) so far away (presumably 10^9 *ly*) that they cannot be registered by optical methods. The radio telescopes have also been able to penetrate into regions (e.g. the galactic core), hidden behind inter stellar dust, which prevent light to pass through.

The main part of the information from space, however, still comes from observatories provided with optical telescopes. The worlds largest mirror telescope was built in 1948 and is found on the Hale observatory Mount Palomar in California. See the figure below.
The hollow mirror in the Hale telescope has a diameter of 5 meters. The telescope is placed in a turnable dome to compensate for the earths rotation, so that the telescope can be directed to a fixed point in space during a long period. Using the Hale telescope there has been taken photos of distant objects and faint object with a exposure time of up to 6 hours.

The Andromeda galaxy, shown in page 1, is an example of a photo taken by the Hale telescope.



After the launching of satellites orbiting above the atmosphere, we have acquired information about the other kind of radiations, we receive from space, which is absorbed in the atmosphere.
Notably a number of X-ray sources have been discovered. Besides the X-ray sources, which can be located to specific objects in space (probably black holes), the space has a constant flow of charged particles.

The particle radiation from space was first detected by R. A. Millikan, who gave it the name *cosmic radiation*. The part of the radiation which enters the atmosphere is called the primary cosmic radiation.
The intensity of the primary cosmic radiation is roughly 1 particle per cm^2 and second.
The secondary cosmic radiation comes about when a primary particle collides with a nucleon in the atmosphere, and in this way creates new particles. After several new collisions with nuclei, one may receive a shower of secondary cosmic radiation coming from one single primary particle.

The main part of the cosmic radiation can be referred to the sun, the so called sun-wind. The sun-wind is particular violent simultaneously with the appearance of sun-spots, which are (nuclear) explosions on the surface of the sun. The other part of the cosmic radiation does not come from the sun, since it is almost isotropic in space.
The remarkable thing about the cosmic particles are their extreme high energies. The particles in cosmic radiation consist mainly of protons and helium nuclei.
The mean energy of the cosmic protons is 10 *Gev*, but particles have been observed with energies up to 10^{11} *GeV*. (The rest mass of the proton is about 1 GeV/c^2). How and where these particles have obtained these extreme energies is still an unsettled question.
The cosmic radiation may incidentally be observed indirectly as “northern light” , which comes about when fast charged particles are refracted in the earths magnetic field near the poles.
In their circular motion they will ionize atoms resulting in radiation of (northern) light.

2. The foundation of astrophysics

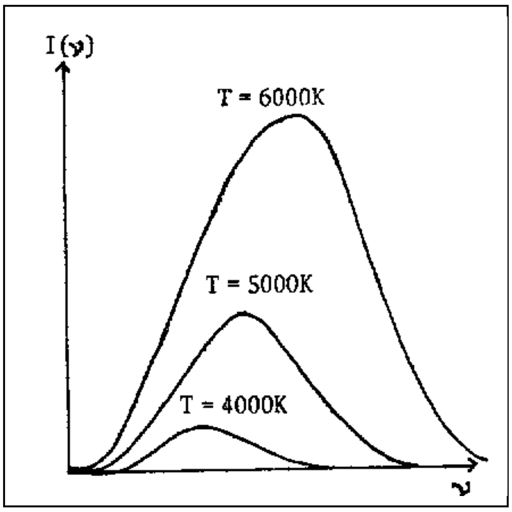
Astrophysics involves many different areas of physics, from which we shall give a summery of the most important.

2.1 Planck’s law of radiation

Planck’s law of radiation is a formula for the intensity dependence of the electromagnetic radiation emitted from a body in thermodynamic equilibrium. Here we just settle for the formula:

Planck’s law of radiation.

(2.1) $$I(\nu) = \frac{8\pi h\nu^3}{c^3(e^{\frac{h\nu}{kT}} - 1)} \quad or \quad I(\nu)d\nu = \frac{8\pi h\nu^3}{c^3(e^{\frac{h\nu}{kT}} - 1)}d\nu$$



The formula should be understood in the manner that $I(\nu)d\nu$ is the fraction of the radiation that is emitted in the frequency interval *dν* (per unit volume of the radiating material). *k* is Boltzmann’s constant, which first appeared in Boltzmann’s formula for the mean kinetic energy of atoms in thermodynamic equilibrium at temperature *T*.

$$< \tfrac{1}{2}mv^2 >= \tfrac{3}{2}kT .$$

Planck’s law of radiation is therefore a statistical, not a deterministic law.
Strictly speaking the formula is only valid when the radiating body is in thermodynamic equilibrium with its surroundings.

This means that the radiating body neither receives nor releases energy.
This is theoretically only possible if the radiating body is confined in a heat isolated black box.
For this reason Planck's law of radiation has gained the rather misleading name (e.g. when the law is applied to the sun): *Black body radiation*.

In astrophysics Planck's law of radiation is applied to determine the surface temperature of a star. From the figure appears that the maximum of the curve is displaced to the right with rising temperatures. In the section on the structure of the atom it is shown that that the frequency ν_{max} where the curve has maximum grows in proportion to the absolute temperature T.
This relation is called Wien's law of displacement.

(2.2) $\nu_{max} = (1.03\ 10^{11}\ Hz/K)\ T$ or $\lambda_{max}\, T = 2.9\ 10^{6}\ nm\ K$

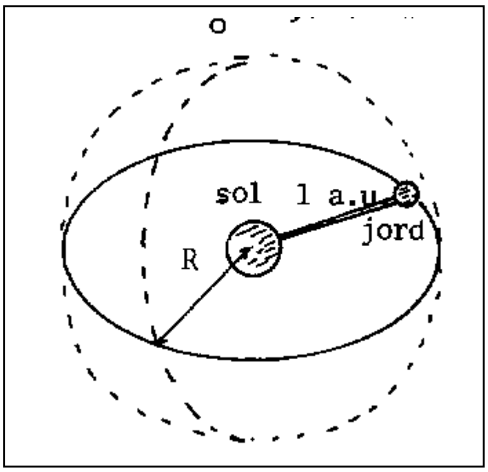
From Wien's law of displacement one can get a survey over the surface temperature of a star and its colour.
Stars with a surface temperature 2000 – 4000 K have λ_{max} in the in the wavelength interval 1450 - 725 nm, and they are therefore red.
The sun has a surface temperature about 6000 K, which gives λ_{max} = 467 nm , and the sun is yellow, as you know.
Stars having a surface temperature more than 9000 K, has λ_{max} less than 320 nm and they are therefore blue white.
From Planck's law of radiation one may also obtain an expression of how the overall radiation from a star depends on temperature. This relation is derived in the section of the structure of the atom.
It is called Stefan-Boltzmann's law. Stefan Boltzmann's law indicates the over all intensity (power per square meter of the surface) emitted from a "black body".

(2.3) $I = I(T) = \sigma T^4$ where $\sigma = 5.67\ 10^{-8}\ W/m^2\ K^4$

2.4 Example.
Using Stefan-Boltzmann's law, we shall calculate the overall flux of energy from the sun, and subsequently demonstrate how the result may be verified experimentally.
The surface temperature of the sun can be determined by recording a spectrum, and then use the displacement law of Wien to find the temperature T_o. = 5800 K. The radius of the sun R_o = 7.0 10^8 m. The total radiated energy is found as the energy flux through a sphere with radius R_o.

$$L_o = 4\pi\, R_o^2 T_o^4 = 4\,\pi\,(R_o = 7.0\ 10^8\ m)^2\ 5.67\ 10^{-8}\ W/m^2\ K^4\ (5800\ K)^4$$

$$L_o = 3.9\ 10^{26}\ W$$



On the earth, one may determine the so called solar constant F_o, which is the intensity of light coming from the sun. (The power received on one square meter placed perpendicular to the incoming light, and corrected for the absorption in the atmosphere) .
The solar constant is the measured calorimetrically to

$$F_o = 1.39\ 10^3\ W/m^2$$

As sketched in the figure the total energy flux from the sun can be found as the solar constant times the area of a sphere with radius 1 AU (1 Astronomical unit, which is the mean distance between the earth and the sun. 1 AU = 1.495 10^{11} m).
$L_o = 4\pi\, R_{AU}^2 F_o = 4\pi\,(1.495\ 10^{11} m)^2\ 1.39\ 10^3\ W/m^2 = 3.9\ 10^{26}\ W$

2.2 Continuous spectra and absorption spectra

The continuous spectrum from a star can be applied to determine the surface temperature.
Planck's law of radiation does not give any information of the radiating body. (Since the law is independent of the nature of the radiating body).

However, the nature of the elements being at the surface of the star can be identified from the *absorption lines* in the continuous spectrum. The absorption lines in the continuous spectrum have been named after the astronomer who was the first to discover them. They are called the *Fraunhofer lines*.
They appear because light with frequencies that match a transition in one of the atoms in the atmosphere are partly removed from the radiation.
This is in accordance with frequency condition of Bohr: $h\upsilon = E_i - E_j$.
So the absorption lines correspond to a difference between two energy levels in an unknown atom, that is, the line spectra of the atom.
However if we limit ourselves to the two elements hydrogen and helium, they have both been identified in the atmosphere of all stars.
But there have never been observed spectral lines that could not be referred to other elements than these we know from earth.

2.3 Where does energy production in the sun come from?

As we calculated it in example (2.4) the energy production in the sun is enormous.
As early as in the mid 1800^{th} it was proposed by Kelvin and Helmholtz that the source of the kinetic energy (heat), was caused by a loss of potential energy when the star contracted.
This should be understood as if a star is considered as an enormous cloud of gas. When it starts to contract, caused by its own gravitation, it is converting potential energy to kinetic energy of the gas atoms.
The relation between the mean kinetic energy of an atom and the temperature T of the gas (in thermodynamic equilibrium) is given by Boltzmann's relation:

(2.5) $$<\tfrac{1}{2}mv^2> = \tfrac{3}{2}kT \quad \text{where} \quad k = 1.38\ 10^{-23}\ J/K$$

A calculation shows, however, that if the present radiation of the sun, should come from gravitation alone, the sun should have an age about 10^7 years. However, this age is 450 times less than the age estimated from other methods e.g. the determination of the age of the earth from the life-time of radioactivity from U-238.
The conclusion is therefore that it is highly unlikely that the energy production from the sun should come from gravitation alone.

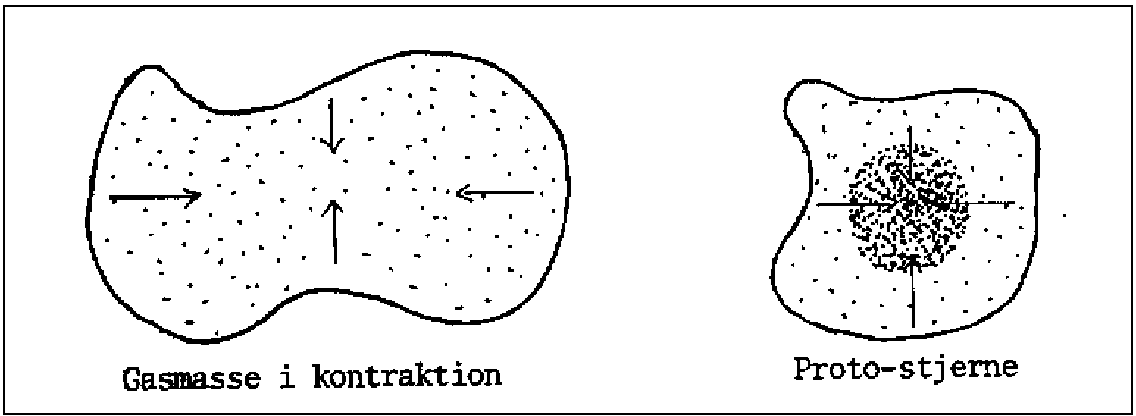
The two physicists H. Bethe and C.F. Von Weitzsäcker suggested in 1930, in the early years of nuclear physics that the energy production of stars could be explained from nuclear processes in the interior of the stars.
The processes should be a fusion of hydrogen to helium, releasing about 26 MeV per reaction.
This figure calculated according to Einstein's equivalence between mass and energy: $\Delta E = \Delta mc^2$.
Since this has been realized, the nuclear physics has played a major role in astrophysics.
The temperatures in the interior of the stars are so high $10^8 - 10^{10}$ K that the atoms are completely ionized. Nuclei and electrons move around in a state of matter that is called plasma.
This has given rise to a new branch of physics called plasma physics.

3. The creation and evolution of a star

In this section we shall give a summary of how we imagine that a star is formed, live and die.
We shall separate it into 4 phases.

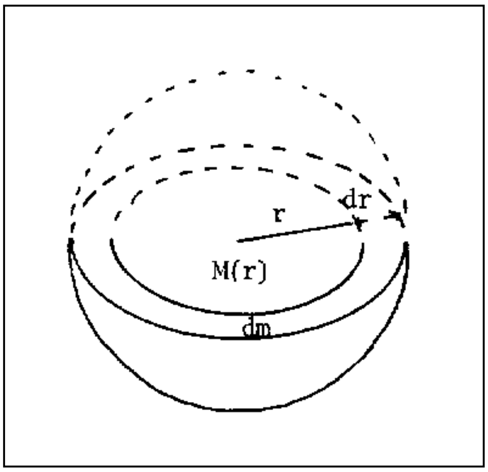
3.1 The contraction phase

In this phase the star is formed by a enormous accumulation of gas starts a contraction because of its gravitational attraction.



3.1 Example

As an example we shall calculate the energy released when a cloud of gas contracts itself and form a body, with the same mass and radius as the sun. This energy is calculated as the difference between the potential energy of the gas cloud, when the gas particles are infinitely apart from each other (potential energy zero) and when they are compressed in a sphere with the same mass as the gas cloud and having a radius equal to that as the sun.



Since the potential energy does not depend on how the sphere is brought together, we shall assume that it is built from thin spherical shells.
In the figure to the left is shown a sphere with radius r and mass $M(r)$, where there is added a spherical shell with mass dm and thickness dr.
The contribution to the potential energy from the shell dE_{pot} may be calculated from the usual formula as the potential energy of a body with mass dm placed at the distance r from $M(r)$.

$$dE_{pot} = -G\frac{M(r)dm}{r}$$

We assume that the sphere is homogenous with density ρ.
The volume of the sphere is $V_{sphere} = \frac{4}{3}\pi r^3$ and the volume of the spherical shell is $V_{shell} = 4\pi r^2 dr$. This gives:

$$M(r) = \tfrac{4}{3}\pi r^3 \rho \quad and \quad dm = \rho 4\pi r^2 dr$$

If we insert these expressions in dE_{pot} , we find:

$$dE_{pot} = -G\frac{(4\pi\rho)^2}{3} r^4 dr$$

Integrating from 0 to R_O.

$$E_{pot} = -G\frac{(4\pi\rho)^2}{3}\int_0^{R_O} r^4 dr \quad \Rightarrow \quad E_{pot} = -\tfrac{3}{5}G\frac{M_O^2}{R_O}$$

To obtain the last expression we have used that: $M_O = \frac{4}{3}\pi R_O^{\,3}\rho$

If we insert the data from the sun: M_O =1.99 10^{30} *kg* and R_O = 7.0 10^9 *m*, the we have: E_{pot} = -2.26 10^{41} *J* .

Assuming that the radiation from the sun has been constant, equal to the present époque: L_O =3.9 10^{26} *W*, then the sun can radiate for a period: $| dE_{pot} |/L_O$ = 5.81 10^{14} *s* = 1.84 10^7 years.

In the calculation above, we have applied some approximations, but it does not change the result that the calculated life-time of the sun (based on the supposition that the radiated energy comes form conversion of gravitational energy to kinetic energy) is about 450 times lower than the established lifetime.

The contraction of the star implies that gravitational potential energy is converted to kinetic energy of the atoms in the stars. Following the contraction the star is heated and it begins to radiate energy, according to Planck's law of radiation and Stefan-Boltzmann's law.
The relation between the temperature of the star and the average kinetic energy of the atoms at the surface of the star is given by Boltzmann's law:

(3.2) $$< \tfrac{1}{2} m v^2 > = \tfrac{3}{2} kT$$

The released gravitational potential energy goes partly to the heating of the star and the rest disappear as radiation. How much goes to heating and how much goes to radiation, appears to be a rather subtle question, but the simple answer is given by the *Virial Theorem*, which states that for a system of mutually interacting particles, the average kinetic energy of the system is the half of the potential energy of the system, and the other half goes to radiation. The virial theorem is normally written as:

(3.3) $$2 < E_{kin} > + < E_{pot} > = 0$$

When the pressure from the upper layers of gas increases, the temperature of the star rises.
This causes, however, a counter pressure from the interior of the star. This pressure may in the first approximation be calculated from the equation of state of ideal gasses. (*P* is the pressure, *V* is the pressure, *N* is the number of particles, *T* is the temperature, *k* is Boltzmann's constant, and *n* is the density of particles)

(3.4) $$PV = NkT \quad \Leftrightarrow \quad P = \frac{N}{V} kT \quad \Leftrightarrow \quad P = nkT$$

In astrophysics the last of the three expressions is the most commonly used.

In the beginning of the contraction phase, the temperature is quite low, and the star is not visible. Gradually as the temperature rises in the interior of the star and the outer layers are also heated. When they reach a temperature of 1000 *K*, the star has become visible.
In spite of the moderate temperature the star will shine quite powerful because of the its large extension, since the total radiation of energy is proportional to the surface.

When the temperature in the interior of the star has reached 10^6 *K*, then the pressure and the temperature are sufficient to initiate a fusion of hydrogen nuclei, and the star enters the *main sequence phase*.

If the mass of the star is less than 0.07 sun masses, then the temperature and the pressure in the star will, however, never be sufficient to initiate nuclear fusion processes. The star will slowly cool off and end as a *black dwarf*.
Thus black dwarfs are not visible, and it has been a, (but now abandoned) hypothesis that the black dwarfs are far more frequent than the visible stars (may still be possible, but hard to verify), and therefore could account for the fact that the mass of the universe seem to be much larger than it is estimated when counting the visible galaxies alone.

The black dwarfs are also called *gas stars*, because their composition probably is the same as in the original universe. They consist of hydrogen and a little helium. But they have no heavy elements, since these elements are created by nuclear reactions in the central partsof the heavier stars.
The planets Jupiter and Saturn are examples of black dwarfs.

3.2 The main sequence phase

If the temperature in the central part of a star exceeds 10^6 K, it is sufficient to start a fusion process of hydrogen to Helium. In this process energy is released, and the temperature in the inner star will increase. Increasing temperature will cause an increasing outward pressure and the contraction of the star will relax.
Hereafter the star will be in a state of equilibrium, where the pressure coming from the outer gas masses of the star will counteract the pressure from the high temperature nuclear reactions in the core of the star.
A contraction of the star will cause an increasing number of fusions, which again will increase the temperature and thereby (from the equation of state of ideal gasses: $P = nkT$) , the pressure will increase. As the pressure raises the volume raises and the density of hydrogen nuclei decreases so the fusion process will be relaxed. The star will expand and become cooler.

In the main sequence phase the star appears as steady shining.

The sun is found in the main sequence phase, and most models for the evolution of stars states that the sun has been in the main sequence phase for 4.5 10^9 years and that it will continue to be in that phase for another 5.2 10^9 years.

The period in which a star is located in the different phases depends on the mass of the star.

The main rule is, however, that the period a star spends in the different phases decreases with its mass. A big star runs through its phases much faster than a light star.

Thus a star having a mass of about times the mass of the sun pass through the main sequence phase in about 2 10^6 years, that is, a thousand times faster than that of the sun.
On the following page is shown a table of the period that stars with various masses will spend in the four phases of their life.

Below is shown the process in which hydrogen nuclei (protons) fusions to helium nuclei.

$$
\begin{aligned}
&{}_1^1H + {}_1^1H \rightarrow {}_1^2D + e^+ + \nu \\
&{}_1^2D + {}_1^1H \rightarrow {}_2^3He \\
&{}_2^3He + {}_2^3He \rightarrow {}_2^4He + 2\,{}_1^1H
\end{aligned}
$$

What we see is the net process is that 4 ${}_1^1H$ nuclei melt together to one ${}_2^4He$ nucleus, emitting two neutrinos and two positrons.

The released energy, the Q-value for the process, is calculated from:

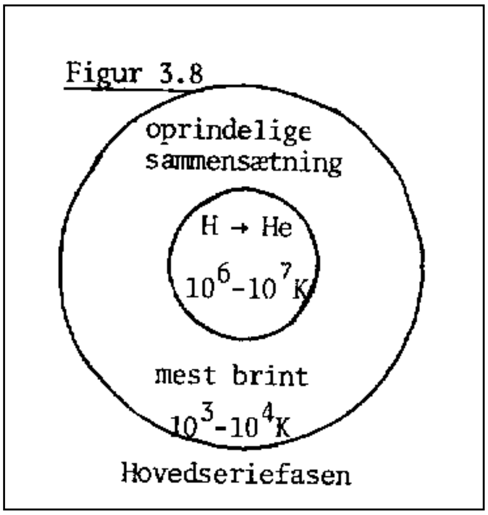
(3.6) $$Q = -\Delta Mc^2 = -(4.00260u - 4 \cdot 1.00783u) = 0.02872u = 26.74 MeV$$

3.7 Example
We have found earlier that the total energy per second radiated from the sun is 3.9 10^{26} *W*.
We shall then calculate how many *kg* of hydrogen per second needs to be burned to helium to maintain this radiation.

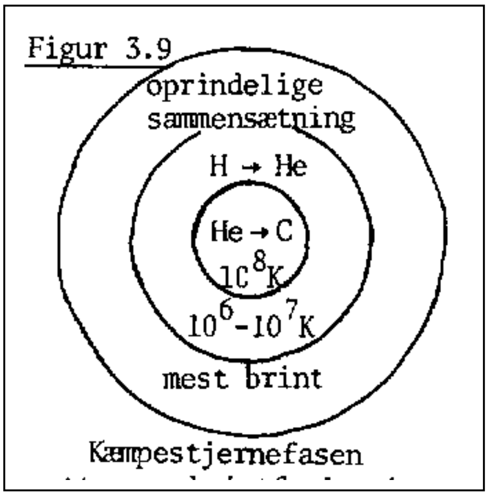
The Q value of the hydrogen fusion is 26.74 MeV = 4.28 10^{-18} *J*. There must therefore be 3.9 10^{26}/4.28 10^{-18} = 9.10 10^{37} fusion processes per second to maintain the energy production. Since there goes 4 hydrogen nuclei into one fusion this corresponds to 3.64 10^{38} hydrogen nuclei. This we shall convert to grams of hydrogen by dividing by Avogadro's number. (1 mole = number of grams of the atomic weight. Avogadro's number = number of atoms in one mole)

The number of hydrogen consumed in fusion per second: $\dfrac{3.64\,10^{38}}{6.023\,10^{23}} = 6.05\,10^{14}\ g = 6.05\,10^{11}\ kg$

The main sequence phase



The giant sequence phase



At the end of the main sequence period, the star consists of a core of helium surrounded by an outer shell having the original composition, mainly hydrogen. The fusion processes in the core will gradually cease, causing the outward pressure to decrease. The star will then again begin to contract and it enters a new phase.

3.4 The giant star phase

At the end of the main sequence phase the star begins a new contraction. In this manner both the density and the pressure increase in the central parts of the star.
The raise of temperature is caused by the release of potential energy, from which half of it goes to kinetic energy and half to radiation according to the virial theorem.

While the central parts of the star becomes smaller and hotter, surprisingly the atmosphere of the star at the time expands and cools. The star has become a *red giant star*.

When the temperature in the central parts has reached 10^8 K, it is enough that a fusion of helium to carbon can be initiated.
This fusion is accompanied by the release of energy, and a new balance between inward pressure from gravity and outward pressure from the rising temperature in the core will be established.
When the fusion of helium to carbon has started the temperature rises to a level where the hydrogen fusion to helium is possible in the layers of the star above the core.

In the red giant star phase we therefore have a central part in which helium fusions to carbon, surrounded by a layer where hydrogen fusions to helium. Finally there will be an outer shell, which has the original composition of matter. This is illustrated in figure (3.9).

The fusion of helium nuclei to carbon is called the triple alpha process. It will proceed as shown below.

(3.10)
$$\begin{aligned} &{}^{4}_{2}He+{}^{4}_{2}He\rightarrow{}^{8}_{4}Be+\gamma \\ &{}^{8}_{4}Be+{}^{4}_{2}He\rightarrow{}^{12}_{6}C+\gamma \end{aligned}$$

The net process is that three ${}^{4}_{2}He$ nuclei melt together in one ${}^{12}_{6}C$ nucleus, accompanied by a release of energy.

The Q-value for the process is: -(12.0000 u-3 4.00260 u) 931 MeV/u = 7.26 u.

In the main sequence phase, we emphasized the balance there is between energy production, temperature and outward pressure. This balance can as an approximation, be described by the equation of state of ideal gasses, according to which the pressure increases in proportion to the temperature, causing the star to expand.

The star has entered *the red giant star phase.*

In the red giant star phase the density in the core of the star is very high, about 10^{10} kg/m^3 . With that density matter do no longer (even approximately) behave as an ideal gas. The gas is said to be degenerated.

In a degenerated gas the pressure no longer grow with temperature, even while the energy production grows significantly because of the rising temperature.
This may cause the fusion of helium to get out of control, and the temperature in the central part of the star will rise dramatically. The star is no longer stable as in the previous phase, and it will result in an explosion, where the outer layers of the star are thrown into space.
This is called a helium flash.
Following the helium flash the star is left only with its central parts, which consists of a small (about the size of the earth) extremely massive ball having a density, about 10^{10} kg/m^3, with temperature (10^4 K – 10^5 K).

The star has become a *white dwarf.*

It is especially a star having the same size as the sun, which ends up as a white dwarf.

In the heavier stars the helium fusion will continue, until the star has a central region of carbon.

If the star does not end in a helium flash, the helium fusion will gradually stop. The energy production and thereby the temperature will drop, and the star will begin a new contraction.
The released gravitational energy from the contraction, will cause a further heating of the central region.
When the temperature reaches 10^9 *K*, it will allow fusion of carbon nuclei to heavier nuclei.
The fusion of lighter to heavier nuclei will continue as long as the fusion melting happens through the release of energy.

As we mentioned in the section of nuclear physics, this will be the case until the atomic number reaches 26 (Iron). The reason for that is that iron and nickel have the highest binding energy per nucleon, so there can be no energetic advances in fusion processes of nuclei with an atomic number above iron.
When the central part of a star has reached a core of iron, the star has no longer any possibility of resisting the gravitational pressure from the outer layers.
What will happen is a violent contraction, a so called collapse and the enormous amount of gravitational energy released, will result in a violent "death" of the star, which we shall deal with in the next section.

3.11 Table.
Of how long a star will stay in the various phases, for some characteristic masses.

Mass	Temperature	Contraction	Main sequence	Giant star
$M_O = 1$	K	Phase 10^6 years	Phase 10^6 years	Phase 10^6 years
0.1	5000	500	10^7	-
0.5	3900	200	10^5	-
1	5800	50	9700	-
3	1400	3	240	90
10	2700	0.2	22	5
30	44000	0.02	4.9	0.8

3.5 The end phase. Death of a star

Stars having a mass less than 1.4 sun masses will end as white dwarfs. As mentioned above, this is caused by the helium flash in the red giant star phase. The outer layers of the star is catapulted out in space in a kind of explosion, and the star is left with is a small hot core, a white dwarf.
A white dwarf has the size of the earth, while the mass is typical that of the sun.
The density is therefore unusually high $10^{10} – 10^{11}$ kg/m^3. In spite of the high temperature of the white dwarfs, they are normally faint caused by their small size.
The most well known white dwarf is Sirius B, which the one of the double star system: Sirius.
It can however only be seen in very powerful telescopes.
In a white dwarf the nuclear reactions have stopped. The star is cooled off only by its radiation of light. The period where the white dwarf shines, can be considerably long about 10^{10} years.

If the mass of the star is larger than two sun masses, then the star will end as a *supernova explosion*, where the outer layers of the star are catapulted out in space. The surface of the star will grow rapidly without a corresponding cooling. The total radiation from the star can be evaluated from the formula:

$$L = 4\pi R^2 \sigma T^4$$

From which we can see that for fixed temperature *T,* the radiated energy, will grow as the square of the radius.
In a supernova explosion however the radius may grow by a factor 10^5 and the brightness of the star will at the same time grow by a factor 10^{10}. This means that the supernova in this period will emit more light than all the 2 billion stars in the Milky way.

For this reason a supernova explosion should not be too hard to observe, but unfortunately they are quite rare (on a scale where human beings have been able to observe them).
Up till 1980 there were only 4 documented rapports on supernova explosions.

Observations from Chinese astronomers indicate that there was one in the year 1006, and one in the year 1054. The Danish astronomer Thyco Brahe observed one in 1648 (Stella nova), and Kepler observed another in 1604. However, the most famous one is the one observed in 1054.

In the 1950-ties astronomers did observations of an expanding cloud of gas in the constellation of the bull. The cloud is called the crab-nebular. Deep in the crab-nebular is located a faint but presumably also a very hot star. (Which can be seen from its blue-white colour)

The observations therefore indicated that the star was the remnant of a supernova explosion.
By taking very accurate photos of the nebular with the period of one or two years, they succeeded (in spite of the distance 5000 *ly*) to determine the speed of expansion of the crab-nebular.
Thereafter it was simple to count backwards to when the supernova explosions must have taken place. They found that it probably had happen about 900 years ago, which fitted nicely with the observations of the Chinese astronomers, both when it concerns time and position in space.

The curious thing is that there presumably are no records from Europe mentioning the phenomenon, although the Chinese records recounts that the explosion was so powerful that it could be seen even at daylight.

Since the atomic elements having an atomic number greater than iron cannot be created by fusion from lighter elements without supplying energy, it is generally assumed that the heavy elements are created in supernova explosions with its enormous outburst of energy.
The building of the heavier elements happens from absorption of neutrons followed by beta-decays which raises the atomic number by one.
The richness of heavy elements on the earth is presumably remnants from a supernova explosions some 5 billion years ago.

As the supernova star explodes, it initiates a collapse of the central parts of the star.
The gravitational forces are so powerful that the contraction continues until the atomic nuclei lies tight to each other and all electrons are pushed back into the nuclei making neutrons.

What is left is a *neutron star*.

A neutron star can be compared with a gigantic nucleus (but without protons).
The mass of a neutron star is 2 – 3 sun masses, but the radius is only about 10 *km*.
The density then becomes the same as density of a nucleon, namely about 10^{18} kg/m^3.
This means that a 1 cm^3 cube would have a weight of 1 million tons.

Because of their small size the neutron stars are so faint that they cannot be observed directly and all intentions of ever observing a neutron star directly had almost been abandoned.
But in 1967 an English astronomer observed a pulsating radio source. It was later verified that the radio waves came from the centre of the crab-nebular. Later several similar radio-sources have been identified.

These radio sources have been named: Pulsars.

The general consensus is that the pulsars are fast rotating neutron stars, which emit radio pulses with the same frequency as in their rotation. Since the frequency of the radio signals from the pulsars is about 100 *Hz* , it means that the neutron stars rotate with the same frequency.

The high rotating frequency of the neutron stars can be explained from the conservation of angular momentum. The angular momentum for a star along an axis is the product of the moment of inertia and the angular velocity: $L = I\omega$. For a sphere: $I = \frac{2}{5}Mr^2$, where r is the radius of the star.
When the star collapses the radius r is dramatically reduced, and since $L = I\omega$ is conserved ω is increased as r^{-2}. So if the star reduces its size by a factor 1000 then the angular momentum and thus the frequency increase by a factor 10^6.

If the mass of a collapsing star, caused by a supernova explosion is bigger than 3 sun masses, the collapse does not stop with a neutron star, but continues because of the enormous gravitational forces. What happens with matter hereafter, we do not know.

But the result of the collapse is called a black hole, and the process is called gravitational collapse.

A black hole is characterized by having a gravitational field that within the horizon of the black hole is so powerful that the escape velocity exceeds the speed of light.

This means that light trying to escape will be sucked back into the black hole. Therefore a black hole cannot emit light or any other radiation and black holes are therefore in principle invisible.

3.13 Example
In mechanics we have derived a formula for the escape velocity from the earth. Using energy conservation we put the energy at infinity equal to the potential and kinetic energy at the surface of the earth. The energy at infinity is zero. The escaping particle has stopped, and the gravitational potential energy is also zero.
If the particle with mass m has the initial velocity v, and the radius of the earth is R, then we may put up the equation:

$$\tfrac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \quad \Rightarrow \quad R = \frac{2GM}{v^2}$$

The last formula gives the radius of a body with mass M , where the escape velocity is v.
If we tentatively (since the formula is non relativistic) put the escape velocity to the speed of light c, then it should give the radius R_S (of the horizon) of a black hole with mass M_S.

$$R_S = \frac{2GM_S}{c^2}$$

It is a remarkable “coincidence”, that this is the same formula that you get from the General Theory of Relativity, by a far more complicated mathematical road.
We could for example find the radius in a black hole with a mass equal to 4 sun masses.

$$R_S = \frac{2 \cdot 6.67\,10^{-11} \cdot 8\,10^{30}}{(3.00\,10^{8})^{8}} m = 1.19\,10^{4}\ m = 11.9\ km$$

The density for a black hole of this size will be: 1.15 10^{18} *kg/m*^{3} the same as the density of a neutron star.

The theory that led to the idea of black holes is Einstein's General Theory of Relativity from 1915, and the mathematical singularity in 4-space that led to the hypothesis of a black hole was shortly afterwards discovered by Schwartzshild. It was, however, not before the 1960-ties that the physical community began to believe that a singularity within a mathematical theoretical framework could actually manifests itself in the physical world.

Also there was a serious more principal problem. Since black holes are invisible, they cannot be observed. Physics is an empirical science, and therefore non observable phenomenon cannot be part of a physical description of the material world.

However in the later years there has been numerous indirect evidence that black holes actually exist, and there are quite abundant in the universe.

One of the first indications of the existence of a black hole was the X-ray source Cygnus X-1.

Cygnus X-1 is a double star system but one of the companions is invisible. It has therefore been suggested that the invisible component could be a black hole. What has been observed is just that the other companion moves in a strong gravitational field.
After the launching of satellites Cygnus X-1, has been identified as a one of the X-ray sources in the universe.
This fits very well into the theory of a black hole sucks gas from the visible companion, the gasses will, caused by the extreme gravitational field obtain that large accelerations that they emit X-Rays before they enter the horizon of the black hole, (behind which no radiation can escape).
In 1975 an outburst of X-ray radiation was observed from Cygnus X-1, but the spectrum was not in very good agreement with theory.

Whereas the existence of black holes was disputed in the sixties and seventies, they are today recognized as an important part of the physical universe.