The Tides
caused by the moon and the sun

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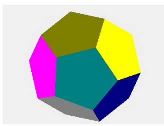


Ole Witt-Hansen 2010

1. What causes the tide?

In grammar school I was taught that the tide was due to the gravitational attraction from the moon. Because of the earths rotation the moons attraction on the oceans is shifted twice within 24 hours. The movement in of the oceans is caused by the (small) difference in the moons gravitational attraction on the side of the earth facing the moon and the side turned away from the moon, usually referred to as the tide.
On the side facing the moon one would expect a bulge, causing high tide, but on the opposite side, one maybe less expected there will also be a bulge caused by the diminished “centripetal” force from the moon, so that the water is “thrown out”.
The latter is often formulated as: “The water is thrown out because of the centrifugal force”. However the centrifugal force is not a physical force but merely reflects the absence of a sufficient centripetal force to keep the body in circular motion.

Since the earth does one round in 24 hours, then according to this simple explanation above, there should high tide twice a day.
The reason that the period is not precisely 24 hours is the moons orbit around the earth.
The moon moves one orbit (360^0) in 27.3 days, and it therefore moves $360^0/27.3 = 13^0.2$ in one day.

One hour in the earths rotation correspond to $360^0/24 = 15^0$.

The period of the tide is therefore 24 h + $13.2^0/15^0$ h = 24 h and 53 min.

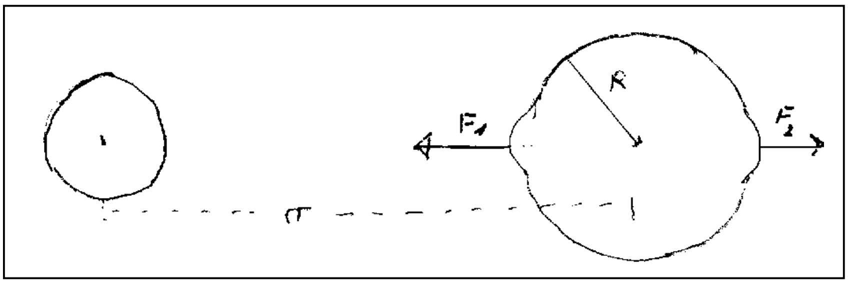
2. The gravitational forces from the moon and the sun

Later when entering the physics stadium in the university, I began to wonder, why the tide was due to the moon only, since the gravitational force from the sun is much larger. But it was only in connection with teaching physics in high school many years later, I found time to do a calculation, that settles the issue. To explain it, we have to deal with Newton’s law of gravitation.

The Gravitational force between two spherical bodies with masses m_1 and m_2, where r is the distance between there centres is:

$$F = G\frac{m_1 m_2}{r^2}$$

G is the gravitational constant: $6.67\ 10^{-11}$ Nm^2/kg^2



It is actually straightforward to set up an expression for the gravitational force from the moon or from the sun on the front side and the back side of the earth.
Let the body that causes the gravitational force have the mass M, acting on a piece of mass m.

R is the radius of the earth, and the distance between the two bodies is *r*. Newton's law of gravitation is then:

$$F(r) = G\frac{mM}{r^2}$$

And next we shall write an expression for the difference in forces on the mass *m* on the front side and on the back side of the earth:

$$\Delta F = G\frac{mM}{(r-R)^2} - G\frac{mM}{(r+R)^2}$$

This we may write as: $\Delta F = F(r-R) - F(r+R)$.
If R << r, then we can apply the approximate formula: $\Delta f = f'(x)\Delta x$, which gives:
$\Delta F = F'(r)\cdot 2R$. Since $F'(r) = -2G\frac{mM}{r^3}$, we get:

$$\Delta F = -2G\frac{mM}{r^3}2R = -4G\frac{mM}{r^3}R$$

We put the mass of the moon to: $M = 7.348\ 10^{22}\ kg$, $R = 6{,}370\ 10^6\ m$, and the distance from the earth to the moon: $r = 60R$. This will cause a difference in acceleration on the two sides.

$$\frac{\Delta F}{m} = 4\cdot 6{,}67\cdot 10^{-11}\frac{7{,}348\cdot 10^{22}}{(60\cdot 6{,}370\cdot 10^6)^3}6{,}370\cdot 10^6 = 2{,}24\cdot 10^{-6}\,m/s^2$$

In grammar school it is most often the moon which is designated as the cause of the tide. However, the gravitational force from the sun is much bigger, and without a calculation one might wonder, why it is not the sun rather than the moon that causes the main contribution to the tide.
The calculation below, however shows, that the sun and the moon contributes almost equally.

The calculation for the contribution from the sun is the same as for the moon, only *M* is now the mass of the sun, and *r* is the distance from the sun to the earth.

Thus we settle for evaluating the ratio between the contribution from the sun and the moon. Index *s* for sun and index *m* for moon:

$$(Moon)\quad \Delta F_m = -4G\frac{mM_m}{r_m^3}R \qquad (Sun)\quad \Delta F_s = -4G\frac{mM_s}{r_s^3}R$$

This gives for the ratio:

$$\frac{\Delta F_s}{\Delta F_m} = \frac{M_s}{M_m}\frac{r_m^3}{r_s^3} = \frac{2{,}0\cdot 10^{30}}{7{,}35\cdot 10^{22}}\left(\frac{3{,}82\cdot 10^8}{1{,}50\cdot 10^{11}}\right)^3 = 0{,}45$$

According to this calculation the contributions to the tide from the sun and the moon are almost equal. (Which is the same result as you find, when consulting a dictionary)

Even if the mass of the sun is about 1000 millions times the mass of the moon, it is the distance to the sun, which enters as the 3. power which gives this (surprising) result.