Newton’s laws

Chapter 4 of the textbook
Elementary Physics 1

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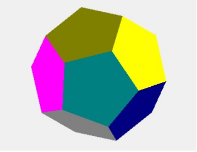

Ole Witt-Hansen 1976 (2017)

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1. The concept of force

When a body is accelerated, you will always (in physics) find that it has a cause.
Newton gave the name forces for the cause of the accelerated bodies.
The reason why an apple falls to the ground is that it is influenced by a force, namely the gravitational attraction from the earth.
When the earth and the other planets orbit around the sun, it is caused by the gravitational attraction from the sun.

It requires a force, when a body is accelerated e.g. a trolley on wheels, but if the runs without friction, it requires no force to maintain a steady motion.

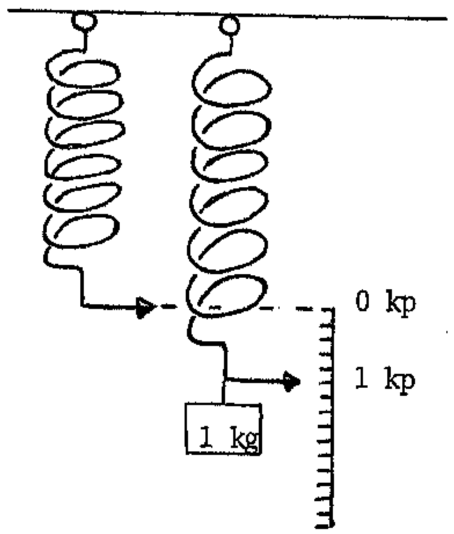
It is one of Newton's greatest merits, that he realized that it does not require a force to uphold a steady motion. Without this assertion, the rest of his work would have been meaningless.

For mechanical experiments on the earth it is impossible to avoid dissipative forces, e.g. a trolley will eventually be brought to rest. The dissipative forces come from friction in the rotating wheels.
If the trolley is turning, however, it requires a force, even if the speed is unchanged.

2. The spring balance as a meter of force

Forces may be measured with a spring balance as shown below, since the elongation (or compression) of the spring is proportional to the force of which it is affected.
The spring balance can be adjusted with mass weights.

Fig. 2.1



The gravitational force that acts on a 1 *kg* weight is called 1 kilopond, and it is written 1 *kp*.
1 pond is therefore the gravitational force on 1 gram.
The force on 2 *kg* is 2 *kp*, and so on.
The spring balance is then adjusted by straining the spring with weights.
The spring balance has the property that the elongation is strictly proportional to the force acting on it, (Hookes law), so if you have marked the positions for 1 kp, 2 kp,…then you can make a equidistant subdivision.
When the spring is used in this manner as force meter it is called a dynamometer.
This rather detailed description is made, because we want to demonstrate, how a daily life concept as force, can be turned into a precise authorized measure of a physical quantity.

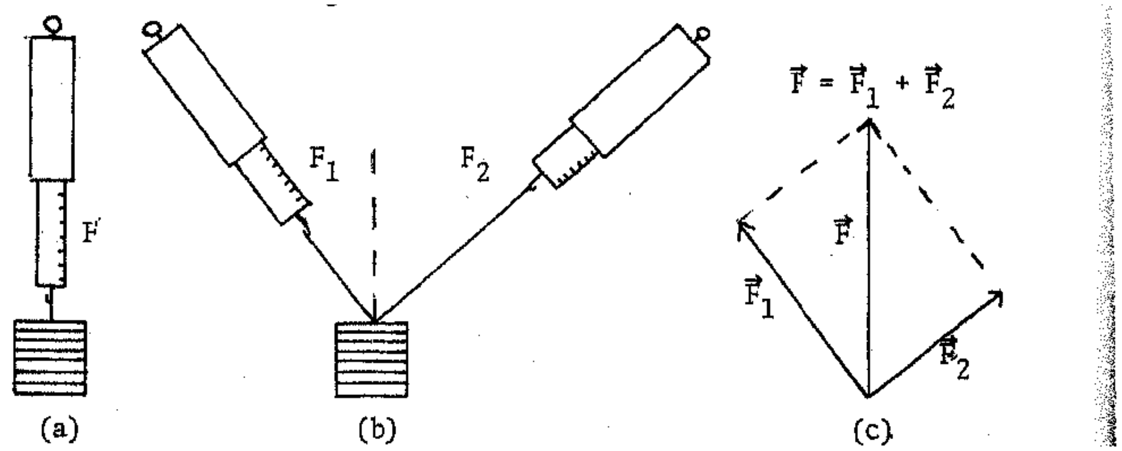
It should be emphasized that the spring balance is in fact a meter of force. If you accelerate a trolley, you can put the spring balance in as an intermediary, where the accelerating force can be directly read.
Nowadays the dynamometer has long gone been replaced by electronic meters, but they are not as illustrating on the principle of measuring a force.,

However, the spring balance, when adjusted on earth, is not universal. If a spring balance is brought to the moon, and is strained with 1 *kg*, then it will only show about 1/6 *kp*.
The mass of the weight is, however, still 1 *kg*. Although mass and force are often used synonymously they are two entirely different physical quantities.
We shall later show how to convert the unit *kp* to *N* (*Newton*), the SI-unit for force.

3. Forces behave like vectors

Obviously a force, like displacement, velocity and acceleration has (besides a numerical value) a direction. Therefore, except for strictly linear motion, a force is written with a vector symbol $\vec{F}$.

In Physics the letter *F* is almost without exception used to denote a force.
That forces actually behave as vectors, it is necessary to establish that a sum of two forces follow the rules of vector addition. Below is shown a simple example that confirms this assertion.



In the figure (a) a weight is suspended in a dynamometer. On the dynamometer the magnitude of the force *F* can be read. $\vec{F}$ denote the force which the dynamometer affects the weight.

In the figure (b), the force $\vec{F}$ is replaced by two forces $\vec{F}_1$ and $\vec{F}_2$, their magnitude can be read on two dynamometers, and their directions are the directions of the cords to the weight.
In the figure (c), all three forces are depicted from the same point, and the experiment will show (with the present accuracy of measurements) that the relation $\vec{F} = \vec{F}_1 + \vec{F}_2$ holds good.
The figure (*c*) is often referred to as the *parallelogram of forces*. $\vec{F}$ is denoted the *resultant* from the two forces $\vec{F}_1$ og $\vec{F}_2$, and $\vec{F}_1$ og $\vec{F}_2$ are denoted the components of $\vec{F}$ along the two directions (1) and (2).
The result of this simple experiment can be formulated as a more general theorem. (3.2)

> If a body is affected by several forces: $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$, The resulting force $\vec{F}$ is found as the vector sum of the given forces: $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3, \ldots$, the effect of $\vec{F}$ can replace all the individual forces, and the body will move as if it was influenced by the resulting force only. This has been proven to hold good, irrespectively of whether the body is moving or is at rest.

Conversely, any physical force $\vec{F}$ can always be resolved after two non parallel directions (1) and (2): $\vec{F} = \vec{F}_1 + \vec{F}_2$. This is applied in the analysis of almost all dynamic situations, as we shall see in the following example.

4. Mass

All extended bodies are in possession of a property called mass. In everyday language mass is often used synonymous with weight, that is, the gravitational force on the body.
As already mentioned the gravitational attraction, varies slightly over the surface of the earth, whereas the mass is an inalterable property that is attached to the body only.

If we do not specify the mass of a body by its weight, it can be characterized by its ability to or its inertness to be accelerated. It is easier to throw a stone weighing 100 *g*, than a stone weighing 2 *kg*.

To understand the difference between weight and mass, you may thing of 3 designs of a football, an ordinary leather ball, one made of wood and one made of iron. If you address each of the three footballs with a well planned kick, then you will (in addition to the comprehensive orthopaedic surgical treatment), certainly sense that the 3 bodies different inertness of being accelerated.

On the other hand if you carried the wooden ball to the moon, then the weight would only about 1/6 of the weight it has on the earth, about the same as the leather ball at earth. However, if you make a kick at it, you will find that it felt exactly the same as it did on earth.
The weight is position dependent, but the mass is not.

As you almost always apply the weight to determine the mass, (and not the inertness to be accelerated), the reason is that it is very easy to measure the weight, and because the (gravitational) weight is proportional to the mass: $F_G = mg$. The gravitational force is directly proportional to the mass. The constant of proportionality being the gravitational acceleration g =9.82 m/s^2.

Using the dish weight, which was still used (also in class) some 50 years ago, you balanced the mass you wanted to determine with some precisely determined weights (down to 1 *mg*). The dish weight, in contrast to the spring balance, gives the same result everywhere, also on the moon.

The modern electronic weights can be applied to the determination of masses that are not too big or not too small. Evidently, the mass of the earth 5.98 10^{24} *kg* or the mass of the hydrogen atom 1.660 10^{-27} *kg* cannot be determined with any human designed weight.
In such cases the masses are determined by the accelerations they get from known forces or delivers to other bodies with which they interact.
For example, the mass of the earth can be determined from Newton's law of gravitation by measuring the acceleration (of gravity) that an apple has when it falls from an apple tree.

The collected mass does not change by any physical or chemical processes. Neither is it changed by heating (burning) or compression.

The law of immutability of the mass was first formulated by the Frenchman Lavoisier, who was the first to invent an accurate dish weight. The problematic issue of the immutability of the mass, was of course, when chemical processes or burning resulted in the creation of gasses.

Lavoisier conducted some very accurate experiments, where he collected the gasses from the processes (which hitherto had been considered as weightless) , and he could conclude, after a long series of measurements that even if different materials reacted with each other and created new chemical elements the collected mass was unchangeable.

The mass is a scalar quantity. It requires only a number and a unit to specify the mass.

The answer to the classical trick question, which one has the greatest weight one *kg* of cotton or one *kg* of lead, the answer is of course the same, namely 1 *kg*.

The difference between the two materials is not their masses but their densities.

When I went to high school we learned (by heart):

That the density is the weight in gram of 1 cm^3 *(cubic centimetre) of the material. This unit of density is still used and is* g/cm^3.

1 cm^3 water weighs 1.00 *g*, so the density for water is 1.00 g/cm^3. 1 cm^3 of iron weighs 7.87 *g*, so the density for iron 7.87 g/cm^3.

Today density is more broadly defined as the mass per unit volume, with the SI unit: kg/m^3.
If the mass *m* has the volume *V*, then the density ρ is defined:

$$\rho = \frac{m}{V}$$

where ρ is the Greek letter rho.

5. Newton's laws

Newton was the first to give a precise definition of the concepts of *force*, *mass* and *acceleration*.
We emphasize that we in the preceding sections have given an independent definition of these concepts and in principle independent methods to measure these three quantities, so that a possible relations between them directly can be verified by experiment, (which is the pumping heart in the development of physics).
Until the beginning of the twentieth century, it was the general view that Newton's laws together with the Maxwell equations for electricity and magnetism was able to deliver the ultimate description of the material world. Today we know, that they does not, but still Newton's laws, and their consequences for the concept of *energy* and *momentum* are the foundation of classical mechanics.

(5.1). Newton's 1. low. The law of Inertia

Any body, which is not influenced by forces (or if the resulting force is zero) is either at rest or moves uniformly along a straight line. No force is required to maintain a steady linear motion

(5.2) Newton's 2. lov: $\vec{F} = m\vec{a}$

The resulting force $\vec{F} = \vec{F}_{res}$ on a body is equal to the mass *m* of the body times the acceleration $\vec{a}$ of the body.

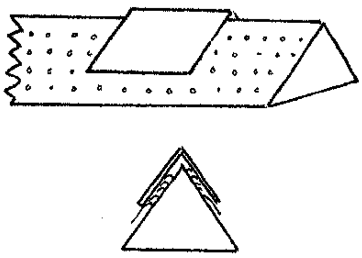
(5.3) Newton's 3. lov: The law about action and reaction

If one body affects another body with a certain force $\vec{F}_1$, then the other body will exert the first body with a equal force $\vec{F}_2$, but having the opposite direction.: $\vec{F}_2 = -\vec{F}_1$, and this holds good, whether the two bodies are at rest or in motion.

6. Newton's 1. law. The law of inertia

General observations on earth do not give an immediate confirmation of the law of inertia. If you give any object with wheels a push on a horizontal underlay, it ill sooner or later come to a halt.

Fig. 6.1



From such observations stems the notion that it requires a force to uphold a motion. (Aristotle).
After Newton's Principia was published it was realized that the braking of a body in motion was due dissipative forces like friction, so that the presuppositions of the law of inertia was not fulfilled.
Doing experiment on the surface of the earth makes it very difficult to avoid dissipative forces.
However after 1970 there has for educational purposes been designed a so called air bench, where the friction is immensely small.
The air bench consists of a track, which is equipped with a lot of tiny holes. To the air bench also belongs a glider, which fits the profile of the track.

An (old fashion) vacuum cleaner presses air in the track from one end and the glider will hereafter rest on a "pillow of air", where after the friction between the track and the glider is vanishing. If you equip the glider with springs in both ends, you can make the glider move back and forth many times after you initially have given the glider a small push.

In the modern age, the law of inertia has been confirmed by launching space probes that move beyond the atmosphere of the earth. Outside in space there is no "air" to cause resistance.

It is important, when formulating the law of inertia, to emphasize the uniform as well as the straight motion. A motion that does not take place along a straight line is always accelerated, even if the speed is constant.

At the same time as Newton stated his 3 laws, he had consideration about their validity, with respect to different observers (coordinate systems), or frames of observation. A frame where Newton's laws are valid is called an inertial frame. Two inertial frames move with constant velocity with respect to each other.

That Newton's laws are actually valid in any inertial system is actually less obvious, since one and the same movement will appear rather different in the two frames.

For example think of a person moving uniform and straight in a train wagon, and by mistake he looses his chain watch, which drops to the floor. From the observer in the train the watch will follow a vertical line and hit the floor vertically below the spot, where it was dropped.

For a stationary observer next to the train, the watch will initially have a horizontal velocity (the same as the train), and therefore this observer will claim that the fall of the watch was not vertical, but rather a (parabolic) trajectory.

Equally important, the example shows that you get the same result of an experiment irrespectively whether it is carried out in a frame "at rest" or in a frame moving straight and uniform, since in both cases the watch will fall down vertically and hit the spot below where it was dropped.

Consequently there is no way to determine, whether a frame is "at rest" or moving straight and uniform, which also is one of the corner stones of The special theory of Relativity.
So the concept "to be at rest" is fictitiously.

Newton's laws and all other laws of nature look the same in all inertial frames.

On the other hand, an accelerated system is not an inertial frame, what you for example can experience in a train that brakes or passes a curve. Objects at rest relative to the compartment, will move or be thrown over, and you may loose your balance, without being able to indicate any forces causing this.

However, if the motions in the compartment of the train are seen from an outsider at rest, then he will find that everything happens in full accordance with Newton's laws.
The surface of the earth is only an approximate inertial frame, since it is accelerated in its orbit around the sun, and it is accelerated in its rotation about its axis.

That the earth is not an inertial frame was first experimentally convincingly demonstrated by the Frenchman Foucault in his attempt to demonstrate that the earth was rotating. He hung a heavy pendulum in a 20 *m* long cord from the upper ceiling in a church, and let it swing. It turned out that the pendulum swing plane turned 360^0 within 24 hours. This would not have happened in an inertial frame, or without the rotation of the earth.

7. Newton's 2. law

Newton's 2. law is the cornerstone in mechanics and the foundation of the rest of the classical mechanics. In its original formulation the law was stated that the resulting force (i.e. the vector sum of forces) that acts on a body is proportional to the mass of the body times its acceleration.

Written mathematically it is expressed: $\vec{F} = c \cdot m \cdot \vec{a}$, where c is a constant that depends on the units used. It is appropriate to choose the units, so that $c = 1$. In SI-units Newton's 2. law therefore takes the familiar form.

(7.1) $$\vec{F} = m \cdot \vec{a}$$

The equation can be conceived as a definition equation for the SI-unit of force.
If the physical quantities on the right hand side are replaced by their units, we find that the SI-unit for force is the SI-unit for mass (*kg*) times the SI-unit for acceleration (m/s^2). The SI-unit for force is called *Newton* and is written 1 *N*. Thus we have:

$$1\ Newton = 1\ N = 1\ kg\ m/s^2 \tag{7.2}$$

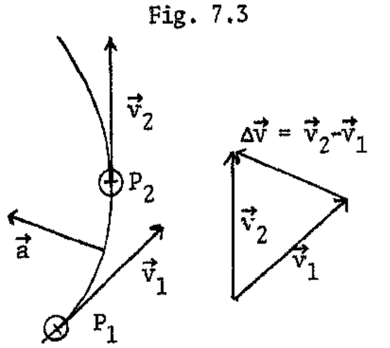
Other units have been in use, notably if the mass is measured in gram (*g)*, the acceleration in (cm/s^2), and then the unit of force becomes $g\ cm/s^2$. This unit is called *dyn*.

$$1\ dyn = 1\ g\ cm/s^2\ (= 10^{-5}\ N).$$

Newton's 2. law is a vector equation, and from this follows that the acceleration always has the same direction as the resulting force.

For a linear motion, the acceleration is always parallel to (same direction or opposite direction) to the force. For a non linear motion this is, however, not the case.

Fig. 7.3



Observing the moon we know that its motion is due to the gravitational attraction from the earth
The force on the moon is therefore always directed towards the centre of he earth, and Newton's 2. law tells us that the acceleration must have the same direction.
In the figure we have (using the rules for subtracting two vectors), tried to illustrate why this is actually the case.
In the point P_1 the velocity is $\vec{v}_1$, while it is $\vec{v}_2$ in the point P_2. The change in velocity is therefore: $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.
In the figure to the right, we have constructed the velocity change $\Delta\vec{v}$ as the difference between the two vectors.
And it is seen that this vector is directed perpendicular to the trajectory.
The acceleration $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ has the same direction, and is therefore directed towards the earth.

7.4 Example
Conversion from the unit *kilopond* (k*p*) to the unit *Newton* (*N)*
Near the surface of the earth all massive bodies fall with the same constant acceleration *g*. If a body has the mass *m* the force on the body is therefore $F_G = mg$. Especially if he mass is 1 kg, then we have earlier defined this force as 1 *kp*. (When the body is at the normal spot in Paris). From Newton's 2. law it then follows:

$$F = mg \quad \Rightarrow \quad 1\ kp = 1\ kg \cdot 9.80665\ m/s^2 = 9.80665\ N$$

Conversely we have $1\ N = 0.1020\ kp$.

7.5 Exemple
A regular block, having the mass $m = 2.0\ kg$ is dragged on a plane underlay with a force meter. There is a certain frictional force between the block and the underlay, but it does not depend on the velocity of the block.
When the block is dragged with constant velocity the force meter shows 4.0 *N*.

a) Find the frictional force F_{gn}.
b) Find the acceleration of the block, when the force meter shows 8.0 *N*.

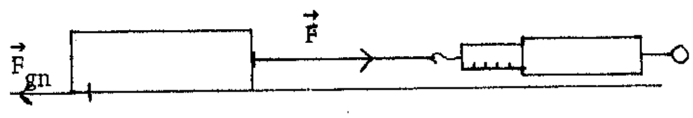
Solution:

For a linear motion, we may skip the vector symbols, but keeping the signs on the quantities.
When the block moves with constant velocity then $a = 0$, so

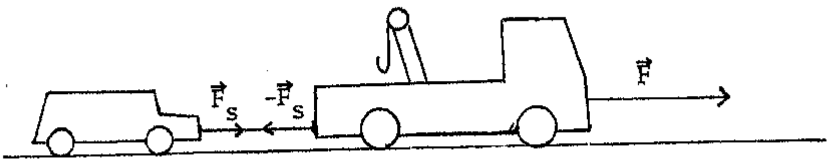
a) $F_{res} = F - F_{gn} = ma = 0 \quad => \quad F = F_{gn} = 4.0\ N.$

b) $F - F_{gn} = ma \quad => \quad 8.0\text{ N} - 4.0\text{ N} = ma \quad => \quad a = 4.0\ N/2.0\ kg = 2.0\ m/s^2.$

Fig. 7.5



A tow truck with the mass M =3.5 tons, must tow a brook down car with the mass m = 1200 kg in a wire.
In this example we shall discard any frictional forces from the wheels.

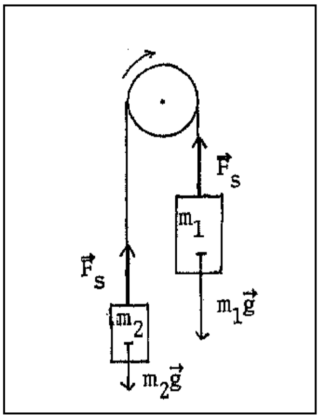


a) Find the force that the truck must deliver to give both cars an acceleration a = 2,0 m/s^2.
b) Find the force F_S in the wire.

Solution:
a) The two vehicles have the same acceleration. From the figure we find the resulting force on the truck:
$F_t = F - F_S = Ma$. The resulting force on the car: $F_c = F_S = m\,a$
Adding the two equations F_S disappears, and we find an expression for the force F generated from the engine of the truck.

$$F = (M + m)a \quad => \quad F = (3500\ kg + 1200\ kg)\ 2{,}0\ m/s^2 = 9400\ N = 95.9\ kp$$
$$F_S = m\,a \quad => \quad F_S = 1200\ kg\ 2{,}0\ m/s^2 = 2400\ N = 24.4\ kp$$

7.7 Example
The figure below is a easy running disc on a shaft (Atwoods machine). An old instrument invented for class experiments with Newton's 2. law. In this case it is provided with two weights having masses m_1 =0.250 kg and m_2 = 0.200 kg. We shall assume that there is no friction, and that it requires no force to rotate the disc. (The line connecting the two weights and the disc is considered mass less).



a) Determine the common acceleration of the two weights.

Solution: Both weights are affected by gravity and by the same force from the line F_S. When the system is in motion, then the two weights have the same acceleration, but with opposite signs. Since when one weight goes up the other goes down..
We then write Newton's second law for each of the two weights..

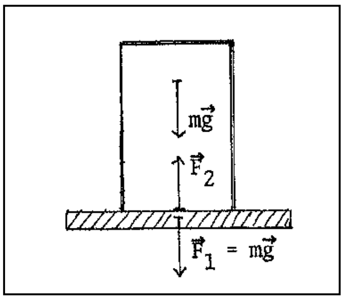
$$F_{res}(1) = m_1 g - F_S = m_1 a_1$$

$$F_{res}(2) = m_2 g - F_S = m_2 a_2 = -\,m_2 a_1$$

If we change sign in the second equation and thereafter add the two equations, we get.

$$m_1 g - F_S = m_1 a_1 \ \wedge\ F_S - m_2 g = m_2 a_1 \quad \Rightarrow \quad (m_1 - m_2)g = (m_1 + m_2)a_1 \ \Leftrightarrow$$
$$a_1 = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \quad \Rightarrow \quad a_1 = \frac{0.050\ kg\ 9.82\ m/s^2}{0.450\ kg} = 1.09\ m/s^2$$

8. Newton's 3. law



8.1 Example
The figure shows a rectangular block placed on a table. If the block has the mass *m* it is affected from gravity by $m\vec{g}$.
Since its acceleration is zero, the resulting force on the block must also be zero, and this implies that there must be another force on the block equal to gravity but with opposite direction. This force is the reaction force to the force with which the block acts on the table.
This is in accordance with Newton's 3. law, which in this case says that if the block acts on the table with a force $\vec{F}_1 = m\vec{g}$ then the table acts on the block which an equal but opposite force $\vec{F}_2 = -\vec{F}_1$.

The two forces $m\vec{g}$ and $\vec{F}_2$ cancel each other, but the action – reaction pair cannot cancel each other, since they act on different bodies.
Any force that acts on a body has according to Newton's 3. law a reaction force. It is the earth that cause the gravitation and therefore the reaction force to $m\vec{g}$ acts in the centre of the earth.

8.2 Example
As it is well established, it is the gravitational attraction, which causes the moon to orbit around the earth. The ratio between the mass of the moon and the mass of the earth is 0.00123.

a) Find the ratio of the accelerations that the earth and the moon are exposed to because of their mutual attraction.
Solution:
According to the third law of Newton, The earth and the moon act on each other with forces, which are equal but opposite. We then write down an expression for these forces, using Newton's 2. law.

$$\vec{F}_{moon} = m_m\vec{a}_m \quad og \quad \vec{F}_{earth} = m_j\vec{a}_j$$

$$\vec{F}_{moon} = -\vec{F}_{earth} \quad \Rightarrow \quad m_m\vec{a}_m = -m_j\vec{a}_j \quad \Rightarrow \quad m_m a_m = m_j a_j \quad \Rightarrow \quad \frac{a_j}{a_m} = \frac{m_m}{m_j}$$

The ratio between the accelerations of the moon and the earth is thus the inverse ratio of the masses. $a_j : a_m$ =0,0123.
For this reason, the smallness the force from the moon, is not something that we are concerned with in daily life.

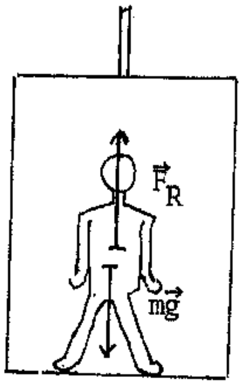
But it should be mentioned that the phenomena of tides is due to the pull in the oceans from the moon, as well as from the sun. From Newton's law of gravitation it follows that the gravitational force from the moon is a little less on the side facing away from the sun, resulting in a different pull in the oceans on the two sides of the earth.

8,3 Example
A person car accelerates on a horizontal road.

a) What is the force that accelerates the vehicle?
b) What is the reaction force to this force, and what generates that force?

Solution:
The force that accelerates the car is the frictional force on which the road acts on the tires of the driving wheels. The reaction force is then the force on which the tires acts on the road. This force comes from the engine through the power transmission.

8.4 Example



A person having the mass 75 kg is standing in an elevator that accelerates up with 1.5 m/s^2

a) Find the force, which the person acts on the floor.

Solution:

The person is affected by gravity $\vec{F}_T = m\vec{g}$, together with the reaction force from the floor. $\vec{F}_R$. According to Newton's 3. law , then $\vec{F}_R$ is equal to but opposite directed to the force we want to calculate. We choose as the positive orientation upwards and write down Newtons's 2. law for the person.

$$\vec{F}_{res} = \vec{F}_T + \vec{F}_R = m\vec{a} \quad \Rightarrow$$

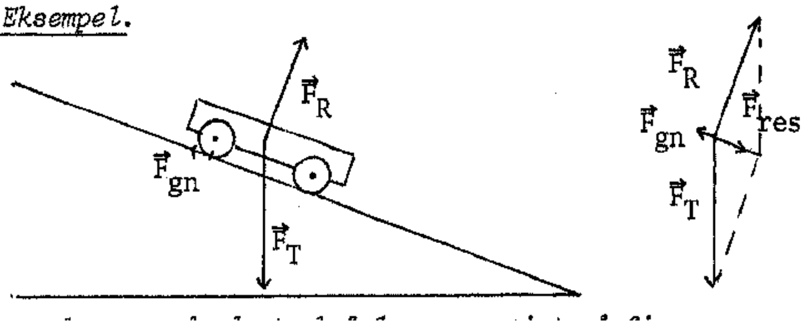
$$F_{res} = -mg + F_R = ma$$

From which we may determine F_R.

$F_R = (a + g)m = (1.5\ m/s^2 + 9.82\ m/s^2)75\ kg = 849\ N,$

Corresponding to a weight of 84 *kg*.

8.5 Eksempel.



A trolley rolls down a plane of inclination, as shown in the figure.

a) Discuss which forces affect the trolley and indicate the direction of the resulting force.

Solution: The trolley is affected by gravity $\vec{F}_T$, the reaction force from the underlay $\vec{F}_R$ and possibly a frictional force $\vec{F}_{gn}$, directed opposite to the motion. The directions of the three forces are depicted in the figure. The resulting force is the vector sum of the three forces: $\vec{F}_{res} = \vec{F}_T + \vec{F}_R + \vec{F}_{gn}$. Since the acceleration is along the inclination, the resulting force has the same direction, as it also appears from the construction of $\vec{F}_{res}$.

8.6 Example

A person car with the mass 1200 *kg* smashes against a concrete wall at a speed of 60 *km/h*. Hereby the car gets a change in velocity from 60 *km/h* to 0 *km/h* at a distance of 0.60 *m*.

a) Find the force on which the car acts on the wall during the collision.

Solution

We assume that the acceleration is constant during the collision. We therefore apply the formula: $v^2 - v_0^2 = 2a(s - s_0)$.

Inserting $v_0 = 60\ km/h = 16.7\ m/s$, $v = 0$ and $s - s_0 = 0.60\ m$, we can calculate the acceleration.

$$a = \frac{v^2 - {v_0}^2}{2(s - s_0)} = \frac{(0 - 16.7\ m/s)^2}{2 \cdot 0.60\ m} = -231\ m/s^2$$

The resulting force on the car is according to Newton's 2. law: $F = ma$ = 1200 *kg* (-231 m/s^2) = -2.77 10^5 *N*.
The force on a passenger having the weight 80 *kg* clamped in a seat belt is correspondingly: 80 *kg* 231 m/s^2 = 1.85 10^4 *N*
Roughly 1900 kp, the weight of about 1.9 tons.
According to the third law of Newton, the force on the wall is equal to but opposite the force on the car.

8.7 Exercises
1. The weight of an elevator is 380 *kg*. What is the force on the lifting wire, when the elevator moves with the acceleration 2,3 m/s^2, up and down respectively?

2. In the adventures of Münchhauser, he pulled himself (and his horse) up from a bog, by a firm pull in his hair.
It is a lie, yes, but why is it impossible?

3. By towing of a wrecked car a wire is applied that has a breaking limit of 290 *kp*. The mass of the car is 1500 *kg*. and the frictional force is estimated to be 10% of the weight of the car.
What is the maximum acceleration the car can endure before the wire breaks.

9. Friction

Dissipative or frictional forces appear everywhere where to bodies are in motion relative to each other. The friction occurs in the surfaces of contact, because the surfaces, when viewed under a microscope, have small irregularities that interact with each other.

Frictional forces can be separated in three groups, having different properties.

1. **Solid material against solid material**: E.g. a wooden block dragged along a horizontal underlay. Experiments show that the frictional force does not depend on the velocity of the block. The laws of frictional forces between to solids are generally simple.

2. Solids against liquid: When a ship sails, there will be a viscous, velocity dependent force between the hull of the ship and the water. Such a force tends to zero, when the velocity goes to zero. The engine (or the sails) is predominantly used to move , that is to accelerate, the headwater that must be supplanted, when the ship moves in the water. The laws related to viscous forces are very complex.

3. Solids against air: A car or a bicycle will, even at moderate speeds be affected by a significant air resistance that in a first (crude) approximation can be assumed to be proportional the square of the velocity.

Common to all frictional and viscous phenomena is that they are always directed opposite to the velocity.

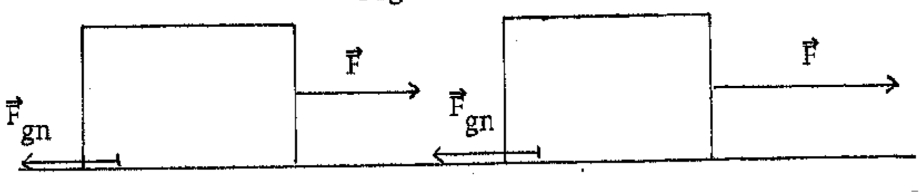
However, in this section, we shall only be occupied with friction between solids.

As an example we shall consider a rectangular block, being dragged on a horizontal underlay.
As long as the block is at rest, the frictional force $\vec{F}_{gn}$ is equal to but opposite to the dragging force $\vec{F}$. ($\vec{F}_{res} = \vec{F} + \vec{F}_{gn} = m\vec{a} = \vec{0}$).

To bring the block in motion, you must act with a force $\vec{F}$, which initially (usually) is greater the frictional force $\vec{F}_{gn}$, when the block is in motion.

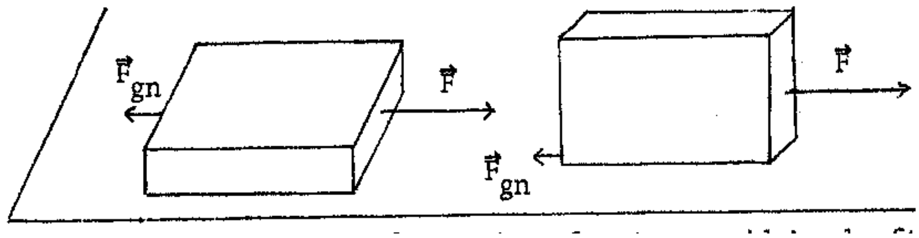
When the block moves, experiments confirm that the frictional force does not depend on the velocity

Fig. 9.1



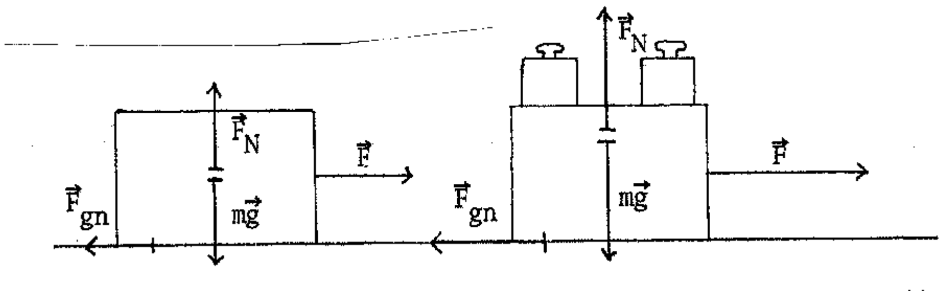
To investigate how the size of the surfaces in contact may influence the frictional force, we may perform an experiment as sketched below.

Fig. 9.2



If the block is dragged in a uniform motion with a force meter, experiments show that the frictional force is the same, irrespectively the block lies flat or it is upright.

The frictional force does not depend on the size of the surfaces in contact, if the surfaces have the same properties moreover.
This can be understood, if you consider that the force per unit area (the pressure) is different, but when you multiply with the area the result is the same, namely the force acting between the two surfaces.



If one strains the block with weights, as shown in the figure above, you will learn that the frictional force increases.
By dragging the block with a force meter, experiments show that the frictional force $\vec{F}_{gn}$ is directly proportional to the force acting between the contact surfaces. This force is called the *normal force* and is denoted $\vec{F}_N$. This can be summarized in the following theorem:

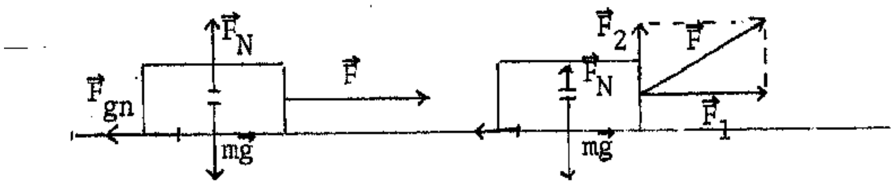
The frictional force when two solid materials move relative to each other, does not depend on the velocity, neither of the size of the contacting surfaces, but only on the normal force $\vec{F}_N$, *as the frictional force is directly proportional to the normal force.*

(9.5)	$F_{gn} = \mu F_N$

μ is a material constant, which is called the frictional coefficient. It depends of the roughness of the surfaces and their nature moreover.
Since μ is a proportional factor between two forces, it does not have a unit.
Concerning friction between two wooden surfaces μ varies from 0.2 to 0.4.

9.6 Example
A rectangular block with mass 2.0 *kg* is dragged over a horizontal with a force on 8.0 *N*.
The frictional coefficient μ is equal to 0.35.
Calculate the frictional force and the acceleration in the two cases:

a) The dragging force *F* is horizontal.
b) The dragging force has the same value, but forms an angle of 30^0 with horizontal.



Solution:

a) In this case $\vec{F}_N = m\vec{g}$, and the frictional force is therefore μmg. From this we find:

$F_{gn} = \mu mg = 0.35 \cdot 2.0\ kg\ 9.82\ m/s^2 \Rightarrow F_{gn} = 6.87\ N.$

The acceleration is then calculated from the resulting force.
...

$$\vec{F}_{res} = \vec{F} + \vec{F}_{gn} \quad \Rightarrow \quad F_{res} = F - F_{gn} = 8.0\ N - 6.87\ N = 1.13\ N$$

$$\vec{F}_{res} = m\vec{a} \quad \Rightarrow \quad a = \frac{F_{res}}{m} = \frac{1.13\ N}{2.0\ kg} = 0.656\ N$$

b) We resolve $\vec{F}$ in a horizontal and a vertical component $\vec{F}_1$ and $\vec{F}_2$.
From the geometry it then follows: $F_2 = \frac{1}{2}F \quad and \quad F_1 = \frac{\sqrt{3}}{2}F$.

In this case the normal force becomes:

$$\vec{F}_N = m\vec{g} + \vec{F}_2 \quad \Rightarrow \quad F_N = mg - F_2 = mg - \tfrac{1}{2}F$$

And we find

$F_N = 2.0\ kg \cdot 9.82\ m/s^2 - 4.0\ N = 15.6\ N.$

And the frictional force becomes

$F_{gn} = \mu F_N = 0.35 \cdot 15.6\ N \Rightarrow F_{gn} = 5.47\ N.$

The acceleration is then calculated from the resulting force.

$$\vec{F}_{res} = \vec{F}_1 + \vec{F}_{gn} \quad \Rightarrow \quad F_{res} = F_1 - F_{gn} = 6.92\ N - 5.47\ N = 1.45\ N$$

$$F_{res} = ma \quad \Rightarrow \quad a = \frac{F_{res}}{m} = \frac{1.45\ N}{2.0\ kg} = 0.725\ m/s^2$$

The example shows that you may obtain a larger acceleration, if you drag the block from an angle, than if you drag it horizontally, which is common knowledge. In this way you ease the normal force and thereby the frictional force. On the other hand you also diminish the dragging force.

You may ask the relevant question of what is the optimal angle of dragging, that is, the angle which gives the largest acceleration for a specific dragging force F.
The answer can be given, but it requires differential calculus, but the angle θ is determined by $\tan\theta = \mu$.

For $\mu = 0.25$ the optimal angle is 14^0, and for $\mu = 0,45$ the optimal angle is 24^0 .