Pressure
Pressure in liquids
Archimedes law

Chapter 5 of the textbook
Elementary Physics 1

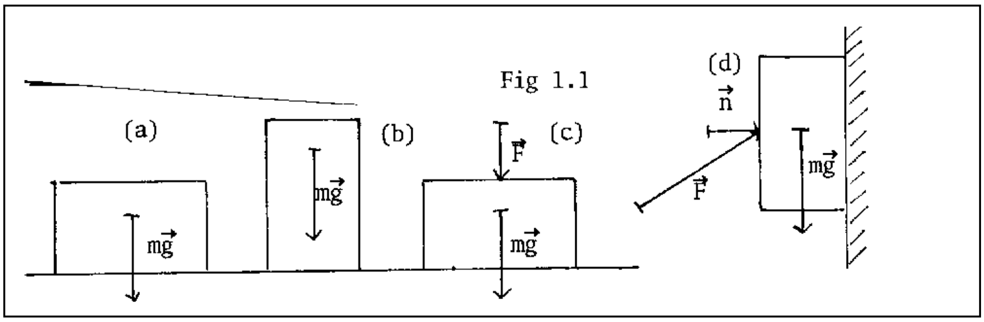
This is an article from my home-page: www.olewitthansen.dk

Ole Witt-Hansen

1977 (2017)

Contents

1. Definition of Pressure



The figures above show different situations where a rectangular block is placed towards an underlay.

In the figure (a) the block is placed on a table. The block is affected by gravity $\vec{F}_T = m\vec{g}$ together with the reaction force from the table $\vec{F}_N$, equal to the normal force, that is, the force perpendicular to the interface.

In figure (b) the block is placed on top, so that the area of the touch surface is smaller.

The force normal to underlay is unaltered $\vec{F}_N = m\vec{g}$, but the force per unit area is greater.

In physics we express this, as the two blocks exert a different *pressure* on the underlay.

We therefore define the pressure against a touch area as the force normal to the surface per unit area. Pressure is either written with a low letter p or a capital letter P.

The pressure is usually considered a scalar, although it has the same direction as the force normal to the surface.

If the force normal to the surface area A is F_N, then we define the *pressure* on the area as:

(1.2)	$p = \frac{F_N}{A}$	(Definition equation of pressure)

From the definition is seen that the SI-unit for pressure is *Newton* per square *meter* (N/m^2). This unit is called Pascal (*Pa*).

In the figure (c) the block is affected by gravity plus an additional force F. So the force normal to the surface is in this case $F_N = mg + F$, and consequently the pressure on the table: $p = \frac{mg + F}{A}$

Gravity has no component perpendicular to the touch area in the figure (d), and therefore it gives no contribution to the pressure on the vertical touch area. The block is held against the wall by a force $\vec{F}$, which forms an angle with the unit normal $\vec{n}$ to the wall.

The force normal to the wall is in this case: $F_N = \vec{F} \cdot \vec{n} = |\vec{F}||\vec{n}|\cos\alpha$ and the pressure against the wall is:

$$p = \frac{\vec{F} \cdot \vec{n}}{A},$$ where A is the touch area.

1.3 Example

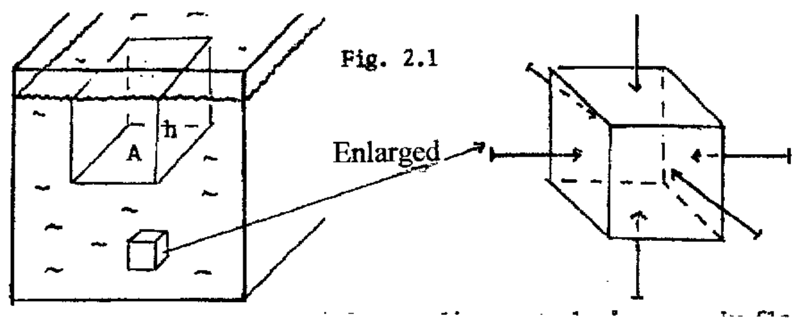
A cylinder shaped weight has the mass 2.0 *kg* and the radius in the circular cross section is 2.0 *cm*.

a) Calculate the pressure the cylinder exerts, when it is placed on a horizontal table.

Solution
The area of the cross section: $A = \pi \cdot r^2 = \pi(4.0\ 10^{-2}\ m)^2 = 5.02\ 10^{-3}\ m^2$.
The normal force: $F_N = 2.0\ kg\ 9.82\ m/s^2 = 19.6\ N$. From which the pressure is determined:

$$p = \frac{F_N}{A} = \frac{19.6\ N}{5.02\,10^{-3}\ m^2} = 3.92\,10^3\ N/m^2$$

2. Pressure in liquids



Pressure in liquids and gasses are defined in the same manner as pressure on a solid surface, as the force normal to the surface per unit area.

In a certain depth of a liquid, the pressure is the same in all directions. Because, if we consider a small cube of liquid at rest, as depicted in the figure, so small that we may ignore its gravity. Then the forces on opposite sides will be equal, since otherwise the cube would move, and the pressure on adjacent sides must also be the same, since otherwise it would be deformed.

Since the pressure is the same in all directions, we simply speak of the pressure in a certain depth. We shall then seek a formula for the pressure p_h in the depth h of a liquid.
We put the density to ρ, and the pressure at the surface of the liquid to p_0.

We then consider a rectangular volume of liquid, where the upper side coincides with the surface of the liquid. The area of that side and of the bottom side is A, and the height (depth) of the rectangular volume is h.
The volume of the box is then $V = A \cdot h$. We then apply the defining equation for pressure:

$$p = \frac{F_N}{A} \quad \Leftrightarrow \quad F_N = pA\,.$$

The force on the upper side is the atmospheric pressure times the area: $F_0 = p_0 A$.
The mass of the liquid in the box is: $m_v = \rho \cdot V = \rho \cdot A \cdot h$
The gravity of the liquid in the rectangular volume is thus: $F_T = m_v g = \rho \cdot A \cdot h \cdot g$.
The normal force on the bottom side of the rectangular volume must be the force normal to that side, which is the force on the upper side plus the gravity of the liquid in the rectangular volume.

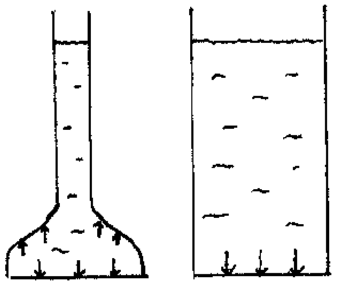
$$F_N = F_T + F_0 = \rho \cdot A \cdot h \cdot g + p_0 \cdot A$$

Since $p = \frac{F_N}{A}$, we can find the pressure in the depth h by inserting F_N and dividing with the area A.

(2.2)	$p_h = p_0 + \rho g h$	(Pressure in the depth h of a liquid)

It is worth noticing that the pressure depends only on the depth, but not on the design of the container, nor on how much liquid, there is in the container.

2.3 Example



The two vessels, shown to the left, have the same ground surface, but different volumes. If they are filled with liquid to the same height, then according to the formula (2.2), the pressure at the two bottoms should be the same. Since the two bottom surfaces have the same area there should also be the same force on the bottom surface!
But how can this be true if the gravity of the liquid in the other vessel is much larger?
It can of course be investigated by placing the two vessels on a weight and the argument above could point towards that the weight would show the same, (which it of course not does!).
So what is wrong with the reasoning? The solution is of course that the weight does not measure the force on the bottom side but the resulting force of gravity.
But for the vessel to the left, the forces from pressure also affects the vessel upwards, forces that must be subtracted from the forces acting from the pressure at the bottom. But it is still correct that the two vessels have the same pressure at the bottom.

3. Units for pressure. Conversions for units

The SI-unit for pressure is, (as already mentioned), *Pascal* (*Pa)* equal to (N/m^2), but especially when gasses are concerned, there are several other units, which have their origins in how the atmospheric air pressure was measured earlier.

3.1 Definition: By the pressure 1 atmosphere, we understand the pressure of a 760 *mm* high quick silver column. To make the conversion to the SI-unit, we apply the formula for the pressure in a liquid with density ρ in the depth h.

$$P(760\ mm\ Hg) = \rho_{Hg} g h = 13.6\ 10^3\ kg/m^3 \cdot 9.82\ m/s^2 \cdot 0.760\ m = 1.013\ 10^5\ Pa$$

(3.2) $$1\ atm = 760\ mm\ Hg = 1.013\ 10^5\ Pa$$

$$1\ mm\ Hg = \frac{1}{760}\ atm = 133.3\ Pa$$

(3.3) (Definition) $1\ Bar = 1\ b = 10^5\ Pa$. $\quad 1\ mb$ (1 *milibar)* $= 10^2\ Pa$

1 *at* is the pressure exerted by 1 *kg* on 1 cm^2.

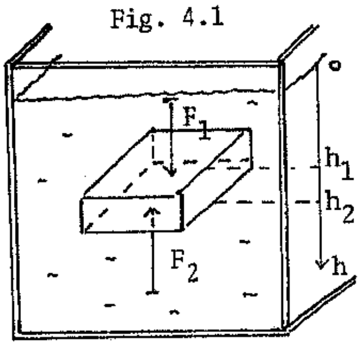
(3.4) $$1\ at = 1\ kp/cm^2 = 9.80665\ N/(10^{-4}\ m^2) = 9.80665\ 10^4\ Pa$$

We can see that 1 *atm*, 1 *Bar* and 1 *at* are almost equal to each other, which do not make it easier. Earlier the air pressure was mostly given in *mb*, and even earlier in *atm*.
Nowadays the air pressure is measured in *hPa* (*hecto-Pascal)*, which is almost the same numerical number as *mb*, a unit the fishermen have used for decades, so they did not really have to make a conversion, listening to the weather forecast. The unit 1 *at* has mostly been used in engineering.

For the pressure in car tires is often used the unit *psi* (pounds per square inch) (also in Europe outside the UK). 1 *psi* =6.895 10^3 *Pa*. Also in daily language pressure is stated as kg/m^2. However, since this it not a physical unit for pressure, then presumably is meant kp/m^2.

4. Archimedes law

Fig. 4.1



The figure shows a rectangular box immersed in a liquid having density ρ.
The pressure on the top side and on the bottom side of the box can be determined from (2.2) $p_h = p_0 + \rho gh$.
The top side is located in the depth h_1, and the bottom side in the depth h_2. Thus we find the pressure on the two sides:

$$p_1 = p_0 + \rho gh_1 \quad \text{and} \quad p_2 = p_0 + \rho gh_2$$

The forces that act on the two sides may be found by multiplying with their common area. A:

$$F_1 = p_1 A \quad \text{and} \quad F_2 = p_2 A.$$

The difference between the forces on the top and the bottom, is called the *buoyancy* and it is denoted F_{up}. We shall then calculate the magnitude of F_{up}.

(4.2) $$F_{up} = F_2 - F_1 = (p_0 + \rho gh_2)A - (p_0 + \rho gh_1)A = \rho g(h_2 - h_1)A$$

The volume V of the box is the height times the ground surface. $V = (h_2 - h_1)A$.
It can therefore hold mass of liquid $m_v = \rho V$. Then we can find an expression for the buoyancy:

(4.3) $$F_{up} = \rho g(h_2 - h_1)A = \rho gV = m_v g \quad \Leftrightarrow \quad F_{up} = m_v g \quad \Leftrightarrow$$

This is Archimedes law:

A body that is immersed in a liquid is affected by a buoyancy which is equal to the gravity of the displaced amount of water..

We have performed a rather detailed explanation of Archimedes law, but only for a rectangular box. However Archimedes law is valid for a body of any shape.
If you (mathematically) confine a volume of liquid exactly the same as the shape of the immersed body, then the liquid is affected by gravity and the pressure from the surrounding liquid. Since it is at rest in the liquid the pressure forces from the liquid must exactly cancel the gravity of the liquid in the volume, which is equal to $m_v g$.

4.4 Exercises

1. a) Find the force that the atmosphere exerts on a 40 x 40 cm^2 seat of a chair.
   b) Is it more or less than the gravity of an elephant?
   c) Why does the chair not crash?

2. What is pressure in bottom of the Pilipino graves, (depth 10.5 *km*)? State the result in *atm*.

3. The following questions should be answered (without hesitation) with a yes or a no.
The buoyancy on a body depends on:

a) The depth where the body is located.
b) The density of the immersed body.
c) Whether it is round or squared but otherwise same volume.
d) The density of the liquid that the body is immersed in.
e) The volume of the immersed body.
f) The pressure on the surface of the liquid (The atmospheric pressure).

4. With a hammer the head of a nail is affected with a force of 400 *N*. The head of the nail has a diameter of 4.0 *mm*, and the tip has a diameter 0.5 *mm*.
Find the pressure on the head of the nail, and at the spot where the nail penetrates.