Everyone has had the experience that, when placed on a rotating disc, one feels an increasing drag towards the edge of the disc.

We shall explore the problem by finding the trajectory that a bullet follows in a rest system, when initially placed at rest on the disc

The problem turns out to be somewhat more complicated than you might think at first sight. The reason for this is that when moving in a rotating coordinate system, you must deal not only with the centripetal force but also with the more intriguing Coriolis force.

For this reason, we shall not analyze the problem in terms of forces, but instead perform a dynamic analyze in an inertial system. (The Lab system). For the motion in an inertial system, there are no fictitious forces, like the centrifugal or the Coriolis force!

The reason for the drag towards the end is frequently eplained as a consequence of the "centrifugal force". The problem with the centrifugal force is, however, that it does not exist. The centrifugal force is a fictitious force that appears as a consequence of moving in a rotating coordinate system, where one cannot apply Newton's laws without restriction.

We chose a coordinate system, having its centre at the axis of rotation. In this system, the position of a fixed object at $\vec{r} = (x, y)$ on the disc is given by the formulas for the uniform circular motion $\vec{r} = (r\cos\omega t, r\sin\omega t)$. An object placed at that point will only be affected by the resulting force, which is the centripetal force: $F_c = m\omega^2 r$. When this force is physically absent, the object will "feel" a radial outward force, that is, the "centrifugal force".
Since this must be true at any instant for any object on the rotating disc, then the only change from the case from the uniform circular motion is that the radius of the object depends on time: $r = r(t)$.
With this assumption, we differentiate the expression for the coordinates: $\vec{r} = (r\cos\omega t, r\sin\omega t)$ twice with respect to time to determinate the instant acceleration. (A small bullet above a variable represents as usual differentiating with respect to time).
However, applying the expression: $\vec{r} = (r\cos\omega t, r\sin\omega t)$, directly makes it rather circumstantial to complete the calculations. Much more convenient is to apply the complex exponential function:

$$\vec{r} = re^{i\omega t} = r\cos\omega t + ir\sin\omega t$$

Differentiating first once and then twice:

$$\dot{\vec{r}} = \dot{r}e^{i\omega t} + ri\omega e^{i\omega t} \qquad and \qquad \ddot{\vec{r}} = \ddot{r}e^{i\omega t} + \dot{r}i\omega e^{i\omega t} + \dot{r}i\omega e^{i\omega t} - r\omega^2 e^{i\omega t}$$

Then we put the acceleration equal to the centripetal acceleration: $\vec{a}_c = \omega^2 re^{i\omega t}$, resulting in the following differential equation.

$$\ddot{r}e^{i\omega t} + 2\dot{r}i\omega e^{i\omega t} - r\omega^2 e^{i\omega t} = r\omega^2 e^{i\omega t} \quad \Leftrightarrow \quad \ddot{r}e^{i\omega t} + 2\dot{r}i\omega e^{i\omega t} - 2r\omega^2 e^{i\omega t} = 0$$

This is a differential equation of second order with constant coefficients, and it can therefore be solved by putting $r = e^{kt}$, where k might be a complex number.

$$k^2e^{kt}e^{i\omega t} + ke^{kt}i\omega e^{i\omega t} - 2e^{kt}\omega^2 e^{i\omega t} = 0$$

By division with $e^{kt}e^{i\omega t}$, we obtain a quadratic equation in *k*. $k^2 + ik\omega - 2\omega^2 = 0$
The discriminator is: $d = -\omega^2 + 8\omega^2 = 7\omega^2$, and thus:

$$k = i\frac{\omega}{2} \pm \sqrt{7}\frac{\omega}{2}$$

We choose the plus sign, since otherwise the motion would be towards the centre of the rotation. We then get:

$$\vec{r} = \vec{r}_0 e^{\frac{\sqrt{7}}{2}\omega t + i\frac{\omega}{2}t} e^{i\omega t} = \vec{r}_0 e^{\frac{\sqrt{7}}{2}\omega t} e^{i(\omega+\frac{\omega}{2})t} = \vec{r}_0 e^{\frac{\sqrt{7}}{2}\omega t} e^{i\frac{3\omega}{2}t}$$

Or written out i coordinates:

$$(x, y) = (r_0 e^{\frac{\sqrt{7}}{2}\omega t}\cos(\tfrac{3}{2}\omega t), r_0 e^{\frac{\sqrt{7}}{2}\omega t}\sin(\tfrac{3}{2}\omega t))$$

This is, by the way, a parametric for the logarithmic spiral. Namely, if we put:

$$s = e^{\frac{\sqrt{7}}{2}\omega t} \Rightarrow \ln s = \frac{\sqrt{7}}{2}\omega t \Rightarrow \omega t = \frac{2}{\sqrt{7}}\ln s \text{, then we find}$$

$$(x, y) = (r_0 s\cos(\tfrac{3}{2}\frac{2}{\sqrt{7}}\ln s), r_0 s\sin(\tfrac{3}{2}\frac{2}{\sqrt{7}}\ln s)),$$

That is, a parametric of the form: $(x, y) = (at\cos(\alpha\ln t), bt\sin(\alpha\ln t))$, being is the most common parametric for the logarithmic spiral.
We also notice that the shape of the graph depends only on the product ωt. If ω is increased with a factor 10, then we obtain the same graph, if we reduce the time with a factor 10.

We shall finish this analysis, by demonstrating the trajectories with some graphs.
The two graphs represent different angular velocities, and different scales, but in fact the two graphs are identical.