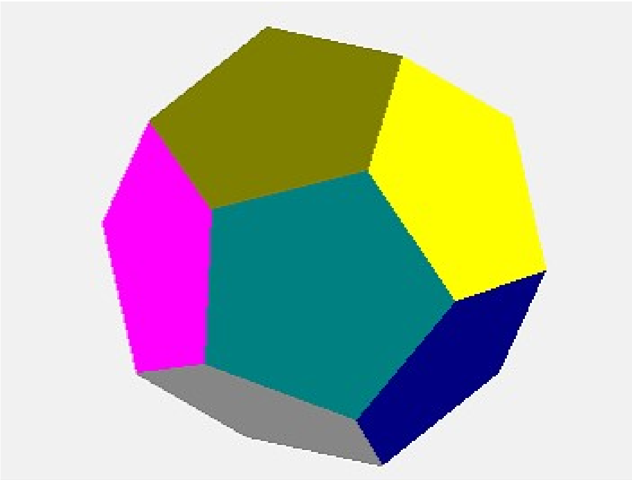
The physics of a hot air balloon



Ole Witt-Hansen 2021 (2018)

1. The buoyancy of a hot air balloon

The buoyancy of a balloon comes either because the balloon is filled with a gas with smaller density then the air, e.g. hydrogen, methane or helium, where the latter, because the former both highly explosive, which helium as a inert gas is not.
Here we shall only be concerned with a hot air balloon, where the atmospheric gas inside the balloon is heated, usually by a gas burner.
When a gas is heated, it expands, and therefore has a lower density, which results in the buoyancy that lifts the balloon from the ground. The difference in pressure inside and outside the balloon is moderate, since the balloon is made of similar light materials as e.g. sails.

But if there was a vacuum in the balloon, there would be an (atmospheric) pressure of $1\ atm = 1.013\ 10^5\ N/m^2$ on the outside of the balloon, corresponding to the weight of $1.013\ 10^4\ kg$, that is, 10,013 *kg* (10 tons) *per.* m^2. This would require that the balloon was built of steel plates of some thickness.

There shall probably be more than 1 *cm* of steel to resist such a pressure, as we shall deal with below. But what is it then that keeps the balloon floating?

The buoyancy on a body is according to Archimedes law equal to:

$$F_{up} = (\rho_1 - \rho_2)Vg\ ,$$

where $(\rho_1 - \rho_2)$ is the difference between the density inside the balloon and in the surroundings. V is the volume of the balloon and g is the gravity.

We shall then proceed to investigate how large the buoyancy is on a hot air balloon, under some simplified assumptions.
We assume for simplicity that the balloon has the shape of a sphere with radius 5.0 *m*.
The volume of a sphere is given by the formula: $V_{sphere} = \frac{4}{3}\pi r^3 = 524\ m^3$.
We assume that outside the balloon the temperature is $20^0\ C$, while the gas in the balloon is supposed to be heated to 60^0 C.
When a gas is heated under constant pressure, we have according to Gay-Lussacs 2. law :

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}\ ,$$

where *T*, is the Kelvin temperature. We then find for the volume after the heating.

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273+60}{273+20} = 1.14$$

This is the factor, by which the volume is increased after the heating.

Since the density is defined as the mass per unit volume: $\rho = \frac{m}{V}$, the density of the gas will be decreased with the same factor. The density of atmospheric air is at 20^0 *C* 1.29 *g/l* = 1.29 *kg/m*^{3}, and the density for the heated gas therefore becomes: $\frac{1.29}{1.14}\, kg/m^3 = 1.13\, kg/m^3$

Hereafter the buoyancy becomes:

$$F_{up} = (\rho_1 - \rho_2)Vg = (1.29 - 1.14)524 \cdot 9.82\ N = 772\ N,$$

which corresponds to the gravity of 78.6 *kg*.

If the diameter is increased to 15 *m*, the buoyancy is increased by a factor 1.5^3 =3,38 and becomes 2609 *N*, corresponding to the weight of 266 *kg*.

An inquirer in the engineers letterbox suggested, that it would be far more advantageous if one used a vacuum balloon. It is straightforward to calculate the buoyancy when the balloon is a steel container with vacuum.

$$F_{up} = \rho_{air} Vg = (1.29)524 \cdot 9.82\ N = 6638\ N \approx 676\ kg$$

Before you raise your arms on this marvellous improvement, you should rather estimate the weight of the steal container, say 1 *cm* steal, which is necessary to hold the vacuum against the outer pressure.

The surface of a sphere with radius r = 5.0 *m* is: $O = 4\pi r^2 = 314\ m^2$, which corresponds to a sphere of steal having thickness 1 *cm*. $\rho_{steel} = 7874\ kg/m^3$, so

$$m_{steel} = 0.1 \cdot 314 \cdot 7874\ kg = 24{,}724\ kg,$$

which is about six times the buoyancy, so the idea of replacing hot air with vacuum is hardly sustainable, so to speak