Contents

800. Determine *a, b* and *c*, such that $2^a + 2^b + 2^c = 2320$

It may be resolved by qualified guesswork, but we shall make it easier, if we divide the equation by 2 as long as it is a even number.

$2320 = 2 \cdot 1160 = 2 \cdot 2 \cdot 580 = 2 \cdot 2 \cdot 2 \cdot 290 = 2^4 \cdot 145$

Since 2320 is the sum of powers of 2, it must be an even number until one of the powers.

If we therefore solve $2^a + 2^b + 2^c = 2320 \quad \Leftrightarrow \quad 2^{a-4} + 2^{b-4} + 2^{c-4} = 145$, one of the powers must be one.

In principle it is the same problem, but the numbers are much easier to handle.

$2^7 = 128$. $145 - 128 = 17 = 16 + 1$, so we have the equation; $2^7 + 2^4 + 2^0 = 128 + 16 + 1 = 145$

And we get the original equation by multiplying by 2^4: $2^{11} + 2^8 + 2^4 = 2048 + 64 + 16 = 2320$

So $a = 11$, $b = 8$ and $c = 4$

801. Determine *a, b* and *c*, such that: $2^a + 4^b + 8^c = 328$

$$2^a + 4^b + 8^c = 328 \quad \Leftrightarrow \quad 2^a + 2^{2b} + 2^{3c} = 328$$

For *a*, *b*, *c* positive 328 must be a power of 2. We therefore successively divide the equation by 2.

$328 = 2 \cdot 164 = 2 \cdot 2 \cdot 82 = 2 \cdot 2 \cdot 2 \cdot 41$ We then have

$$2^{a-3} + 2^{2b-3} + 2^{3c-3} = 41 = 32 + 8 + 1 = 2^5 + 2^3 + 2^0 \quad \Rightarrow \quad a - 3 = 5, 2b - 3 = 3 \ \ and \ \ 3c - 3 = 0 \quad \Rightarrow$$
$$a = 8, \ b = 3, \ \ c = 1$$

802. Solve for *x*: $\left(\frac{1}{x}\right)^x = 4^{x + \frac{1}{16}}$

$\left(\frac{1}{x}\right)^x = 4^{x + \frac{1}{16}}$ The equation cannot be solved analytically, but $x = \frac{1}{16}$ seem to do the trick, since:

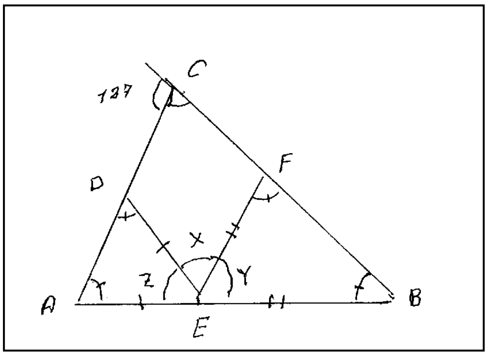
$$\left(\frac{1}{\frac{1}{16}}\right)^{\frac{1}{16}} = 4^{\frac{1}{16} + \frac{1}{16}} \quad \Leftrightarrow \quad 16^{\frac{1}{16}} = 4^{\frac{2}{16}} \quad \Leftrightarrow 16^{\frac{1}{16}} = (4^2)^{\frac{1}{16}} = 16^{\frac{1}{16}}$$

803. Solve for *x*: $x \cdot 3125^x = 1$

$3125 = 5 \cdot 625 = 5 \cdot 25^2 = 5^5$ So we have: $x \cdot 5^{5x} = 1$ And it sees to have the solution $x = \frac{1}{5}$ since

$$\frac{1}{5} \cdot 5^{5 \cdot \frac{1}{5}} = 1 \quad \Leftrightarrow \quad \frac{1}{5} \cdot 5 = 1$$

804. Determine the angle *X* in the figure below.



From the figure we may establish some relations

1. $C = 180^0 - 127^0 = 53^0$
2. $A + B = 180 - C = 127^0$
3. Since ΔEFB is isosceles have: $F = B$
4. Since ΔAED is isosceles we have: $A = D$
5. $X + Y$ is supplementary angle to $Z = AED$, so $X + Y$
   $X + Y = 180 - Z = A + D = 2A.$
   $X + Y = 2A.$
6. $X + Z$ is supplementary angle to $Y = BEF$, so
   $X + Z = 180 - Y = B + F = 2B$
7. The sum of the angels in $DCFE$ is 360^0. So
8. $X + 180 - D + 180 - F + 53 = 360 \Rightarrow$
   $X - A - B + 53 = 0$

We thus have the equations:

1. $A + B = 127^0$ 2. $X + Y = 2A$ 3. $X + Z = 2B$ 4. $X + 53 = A + B$ $X + Y + Z = 180$

From the fourth equation we find: $X = A + B - 53 = 127 - 53 = 74$.
But could also be found 1+2+3:

$$2X + Y + Z = 2(A + B) \Leftrightarrow X + 180 = 254 \Leftrightarrow X = 254 - 180 = 74$$

Since we have 5 unknowns B, C, X, Y, Z but only 3 independent equations it is in principle not possible to determine them all.

805. Determine *x* from: $x^{21} + x^{14} = 36$

. $x^{21} + x^{14} = 36 \Leftrightarrow (x^7)^3 + (x^7)^2 = 36$

We put $x^7 = y$ $(y)^3 + (y)^2 = 36$, which is seen to have the solution $y = 3$, since $3^3 + 3^2 = 36$,
So the solution is $x = \sqrt[7]{3}$

804. Find the value of the integral $\int_0^{\frac{\pi}{2}} \frac{x}{\tan x} dx$

We call the integral: $\int_0^{\frac{\pi}{2}} \frac{x}{\tan x} dx$ for (A)

$$\int_0^{\frac{\pi}{2}} \frac{x}{\tan x} dx = \int_0^{\frac{\pi}{2}} x d \ln(\sin x) = \left[x \ln(\sin x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \ln(\sin x) dx = -\int_0^{\frac{\pi}{2}} \ln(\sin x) dx$$

We call the integral: $\int_0^{\frac{\pi}{2}} \ln(\sin x) dx$ for (B) , and we therefore have A = -B

There is a theorem: $\int_0^a f(x)dx = \int_0^a f(a-x)dx = \left[-F(a-x)\right]_0^a = -(F(0)-F(a)) = F(a)-F(0)$

We make the substitution: $x = \frac{\pi}{2} - u$ in B. This gives:

$$\int_0^{\frac{\pi}{2}} \ln(\sin x)dx = -\int_{\frac{\pi}{2}}^0 \ln(\sin(\tfrac{\pi}{2}-u)du = -\int_{\frac{\pi}{2}}^0 \ln(\cos u)du = \int_0^{\frac{\pi}{2}} \ln(\cos u)du = \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = \text{B:}$$

Then we add: $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx + \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = 2\text{B}$

$$2B = \int_0^{\frac{\pi}{2}} \ln(\sin x)dx + \int_0^{\frac{\pi}{2}} \ln(\cos x)dx = \int_0^{\frac{\pi}{2}} (\ln(\sin x) + \ln(\cos x))dx = \int_0^{\frac{\pi}{2}} \ln(\sin x \cos x)dx$$

And make use of the logarithmic rule for the integrands $\ln(ab) = \ln a + \ln b$: $\sin 2x = 2\sin x \cos x$

$$2\text{B} = \int_0^{\frac{\pi}{2}} \ln(\sin x \cos x)dx = \int_0^{\frac{\pi}{2}} \ln(\tfrac{1}{2}\sin 2x)dx \text{ followed by } u = 2x$$

$$2\text{B} = \tfrac{1}{2}\int_0^{\pi} \ln(\tfrac{1}{2}\sin(u)du = \tfrac{1}{2}\int_0^{\pi} \ln(\sin(u)du - \tfrac{1}{2}\left[x \ln\tfrac{1}{2}\right]_0^{\pi} = \tfrac{1}{2}\int_0^{\pi} \ln(\sin(u)du - \tfrac{\pi}{2}\ln 2 = \tfrac{1}{2}\int_0^{\pi} \ln(\sin(x)dx - \tfrac{\pi}{2}\ln 2$$

Since sin is an even function: $\sin(\pi - x) = \sin x$, we have: $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_{\frac{\pi}{2}}^{\pi} f(\sin x)dx$

$$2\text{B} = \tfrac{1}{2}\int_0^{\pi} \ln(\sin(x)dx - \tfrac{\pi}{2}\ln 2 = \tfrac{1}{2}\cdot 2\int_0^{\frac{\pi}{2}} \ln(\sin(x)dx - \tfrac{\pi}{2}\ln 2 = \int_0^{\frac{\pi}{2}} \ln(\sin(x)dx - \tfrac{\pi}{2}\ln 2 = B - \tfrac{\pi}{2}\ln 2$$

$$B = \int_0^{\frac{\pi}{2}} \ln(\sin x)dx = -\tfrac{\pi}{2}\ln 2 = -A$$

$$\int_0^{\frac{\pi}{2}} \frac{x}{\tan x}dx = \tfrac{\pi}{2}\ln 2$$

805. Determine the value of i^i

$$i^i = e^{\ln i^i} = e^{i \ln i}$$

So we have to find an expression for $\ln i$. We put:

$$ix = \ln i \quad \Leftrightarrow \quad i = e^{ix} \quad and \quad e^{ix} = cox + i\sin x \quad \Rightarrow \quad e^{ix} = i \quad \Rightarrow x = \frac{\pi}{2}, so \quad \ln i = i\tfrac{\pi}{2}$$

$$i^i = e^{i \ln i} = e^{i \cdot i\frac{\pi}{2}} = e^{-\frac{\pi}{2}}$$

806. Solve for *x*: $\left(\frac{1}{x}\right)^x = e^{\frac{\pi}{2}}$

There is no really analytical solution to such an equation, but

Since $i^i = e^{i\ln i} = e^{-\frac{\pi}{2}}$, we "guess" that $x = -i$;

$\left(\frac{1}{x}\right)^x = x^{-x}$, so we calculate

$(-i)^{-(-i)} = e^{i\ln(-i)}$ So we must find $\ln(-i)$

$\ln(-i) = iy \quad \Leftrightarrow \quad e^{iy} = -i$ and $e^{iy} = \cos y + i\sin y = -i \quad \Rightarrow \quad y = -\frac{\pi}{2} \qquad \ln(-i) = iy = i(-\frac{\pi}{2})$

$x^{-x} = (-i)^i = e^{i\ln(-i)} = e^{i\cdot i(-\frac{\pi}{2})} = e^{-\frac{\pi}{2}}$

806. Solve for *x*: $\left(\frac{1}{x}\right)^x = e^{\frac{\pi}{2}}$

807. $\frac{1}{11}+\frac{2}{13}+\frac{4}{17}=x$**. Determine** $\frac{8}{11}+\frac{7}{13}+\frac{5}{17}=y$

The problem is not formulated as if we should determine *x*, but rather express *y* by *x*,
Well:

$x+y=\frac{1}{11}+\frac{2}{13}+\frac{4}{17}+\frac{8}{11}+\frac{7}{13}+\frac{5}{17}=\frac{9}{11}+\frac{9}{13}+\frac{9}{17}=9(\frac{1}{11}+\frac{1}{13}+\frac{1}{17})$. So $y=9(\frac{1}{11}+\frac{1}{13}+\frac{1}{17})-x$

$(\frac{1}{11}+\frac{1}{13}+\frac{1}{17})=\frac{13\cdot17+11\cdot17+11\cdot13}{11\cdot13\cdot17}=\frac{551}{11\cdot13\cdot17}=\frac{29\cdot19}{11\cdot13\cdot17} \quad y=9\frac{29\cdot19}{11\cdot13\cdot17}-x$

808. Solve the integral equation: $\left(\frac{dy}{dx}\right)^2 - 1 = x^2$

$\left(\frac{dy}{dx}\right)^2 - 1 = x^2 \quad \Leftrightarrow \quad \left(\frac{dy}{dx}\right) = \sqrt{x^2+1}$

We make the substitution: $x = \cosh t \quad \Rightarrow \quad dx = \sinh t dt$

$dy = \sqrt{x^2+1}\,dx \quad \Leftrightarrow \quad dy = \sqrt{\cosh^2 t + 1}\sinh t dt = \sinh^2 t dt$

$\int dy = \int \sinh^2 t dt dt$

We have the formula;

$\cosh 2x = 1 - 2\cosh^2 x = 2\sinh^2 x - 1 \quad \Rightarrow \quad \cosh^2 x = \frac{1+\cosh 2x}{2} \quad and \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}$

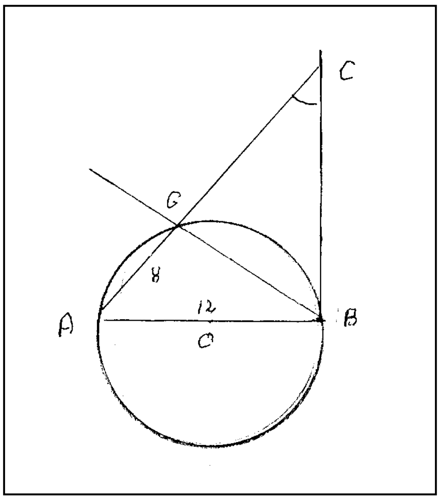
So we have: $y = \int \sinh^2 t dt = \int \frac{\cosh 2t - 1}{2} dt = \frac{\sinh 2t}{4} - \frac{t}{2}$

And substitution back $x=\cosh t$:

$$\sinh 2t = 2\sinh t\cosh t = 2\sqrt{\cosh^2 t - 1}\,\cosh t = 2x\sqrt{x^2-1}$$

$$y = \frac{x\sqrt{x^2-1}}{2} - \frac{\cosh^{-1} x}{2} + c$$

809. Determine the sides and angles in the triangle below



Since the angle G spans an angle of 180^0 then $G = 90^0$.
Since $B = 90^0$ we have $C = 90^0 - A$.
From the trigonometric formulas for a right angle triangle, we have

From the right angle triangle AGB we have: $\cos A = \frac{8}{12} = \frac{2}{3}$ and

$$\tan A = \frac{\sin A}{\cos A} \Rightarrow \tan^2 A = \frac{\sin^2 A}{\cos^2 A} = \frac{1-\cos^2 A}{\cos^2 A} \Leftrightarrow$$

$$\tan^2 A = \frac{1-(\frac{2}{3})^2}{(\frac{2}{3})^2} = \frac{5}{4} \Rightarrow \tan A = \frac{\sqrt{5}}{2} \Rightarrow A = 48.19$$

$$C = 90 - A = 51.81$$

$$\tan A = \frac{|BC|}{|AB|} \Rightarrow |BC| = |AB|\tan A = 12\frac{\sqrt{5}}{2} = 6\sqrt{5}$$

$$|AC|^2 = |AB|^2 + |BC|^2 = 144 + 180 = 324 \Rightarrow |AC| = 18$$

810. Solve for x. $x^x = 3^{1215}$

Well; $1215 = 5\cdot 243 = 5\cdot 3\cdot 81 = 5\cdot 3\cdot 3^4 = 5\cdot 3^5$ So we have: $1219 = 3^{5\cdot 3^5} = (3^5)^{3^5}$ so $x = 3^5$

811. Solve for x: $x^{x^4} = 64$. It seems that $x = \sqrt[4]{8}$ does the trick, since:

$$(\sqrt[4]{8})^{\sqrt[4]{8}^4} = (\sqrt[4]{8})^8 = 8^2 = 64$$

812. Solve the differential equation: $\frac{dy}{dx} = y + x^2 \Leftrightarrow \frac{dy}{dx} - y - x^2 = 0$

This is a homogenous differential equation of 1. degree, and it is solved by multiplying the equation by $\exp(\int -1dx)$

$(ye^{-\int dx})' = y'e^{-\int dx} + y(-1)e^{-\int dx}$ We thus have: $(ye^{-\int dx})' - x^2 e^{-\int dx} = 0 \Leftrightarrow ye^{-x} = \int x^2 e^{-x} dx$

$$\int x^2 e^{-x} dx = - - \int x^2 de^{-x} dx = -x^2 e^{-x} + \int 2xe^{-x} dx$$

$$\int 2xe^{-x} dx = -\int 2xde^{-x} = -2xe^{-x} + \int 2e^{-x} =$$

$$\int 2\int 2e^{-x} = -2e^{-x}$$

$ye^{-x} = \int x^2 e^{-x} dx = -x^2 e^{-x} + -2xe^{-x} + 2e^{-x} + C$ And the solution is then:

$y = -x^2 - 2x + 2$

813. solve the differential equation $\frac{dy}{dx} = \frac{y^2+1}{y^3}$

$$\frac{dy}{dx} = \frac{y^2+1}{y^3} \Leftrightarrow \frac{y^3}{y^2+1}dy = dx \Leftrightarrow \frac{y(y^2+1)-y}{y^2+1}dy = dx \Leftrightarrow \frac{y(y^2+1)-y}{y^2+1}dy = dx \Leftrightarrow$$

$$(y - \frac{y}{y^2+1})dy = dx \Rightarrow \int (y - \frac{y}{y^2+1})dy = x \Leftrightarrow \tfrac{1}{2}y^2 - \tfrac{1}{2}\ln(y^2+1) + c$$

814. Determine *a* and *b* from: $a^3 + b^3 = 10$ and $a^2 + b^2 = 7$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a+b)^3 = 10 + 3ab(a+b)$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)^2 = 7 + 2ab$

$(a+b)^3 = (a+b)^2(a+b) = (7+2ab)(a+b) = 7(a+b) + 2ab(a+b) = 10 + 3ab(a+b)$

$ab(a+b) = 7(a+b) - 10$

$(a+b)^3 = 10 + 3ab(a+b) = 10 + 3\cdot(7(a+b) - 30) \Rightarrow (a+b)^3 = 21(a+b) - 20$

We put $u = (a+b)$ to give;

$u^3 = 10 + 21u - 30 \Leftrightarrow u^3 - 21u + 20 = 0 \Leftrightarrow$

We can immediately see that $u = 1$ is a root, since $1^3 - 21\cdot 1 + 20 = 0$

And we therefore make polynomial division with u -1

$u - 1 | u^3 - 21u + 20 | u^2 + u - 2$

$u^3 - u^2$

$u^2 - 21u$

$u^2 - u$

$-21u + 20$

$-21u + 20$

$$u^2 + u - 20 = 0; \quad d = 1 + 4\cdot 20 = 81 = 9^2; \quad u = \frac{-1 \pm 9}{2} = \begin{cases} 4 \\ -5 \end{cases}$$

$a + b = 4 \quad or \quad a + b = -5$

$a+b=4 \quad and \quad ab(a+b)=7(a+b)-10 \quad \Rightarrow \quad ab=\dfrac{9}{2}$

$a+b=-5 \quad and \quad ab(a+b)=7(a+b)-10 \quad \Rightarrow \quad ab=9$

$a+b=4 \;\; and \;\; ab=\frac{9}{2} \quad \Rightarrow \quad ab+\dfrac{9}{2}=4 \quad \Rightarrow \quad 2a^2+9=8a \quad \Leftrightarrow$

$2a^2-8a+9=0; \;\; d=64-72 \; <0$

$a+b=-5 \;\; and \;\; ab=9 \quad \Rightarrow \quad a+\dfrac{9}{a}=-5 \quad \Rightarrow \quad a^2+5a+9=0 \;\; ; \;\; d=81-36=45$

$a=-9\pm\sqrt{45} \quad and \quad b=\dfrac{9}{a} \;=\; \dfrac{9}{-9\pm\sqrt{45}}$

815. Determine *x* from the equation: $x^{\log 25}+25^{\log x}=10$

We notice the rule $a^{\log b}=b^{\log a}$ since $\log a^{\log b}=\log b\log a \;\; and \;\; \log b^{\log a}=\log a\log b$, so

$x^{\log 25}=25^{\log x} \quad \Rightarrow \quad x^{\log 25}+25^{\log x}=10 \quad \Leftrightarrow \quad 2\cdot 25^{\log x}=10 \quad \Leftrightarrow \quad 25^{\log x}=5 \quad \Leftrightarrow$

$\log x\log 25=\log 5 \quad \Leftrightarrow \quad 2\log 5\log x=\log 5 \quad \Leftrightarrow \quad \log x=\frac{1}{2} \quad x=10^{\frac{1}{2}} \quad x=\sqrt{10}$

816. Solve for *x* : $e^x-1=\ln(x+1)$

The solution is seen to be $x = 0$, and that is the only, solution since $e^x>\ln(x+1)\; for \;\; x>0$

817. Prove that $|AO|\cdot|OC|=|BO|\cdot|OD|$

Where AB and DC are two intersecting secants in a circle, and *O* is their intersecting point.
We draw the lines *AB* and *CD,* $\angle ABD$ *and* $\angle ACD$ are peripheral angles that span the same arc on the circle, so they are equal. *ABO* and *DCO* are equiangular triangles. So the ratio between equilateral sides is the same. So we may write:

$$\frac{|AB|}{|CD|}=\frac{|AO|}{|DO|}=\frac{|BO|}{|CO|} \quad \Rightarrow \quad |AO|\cdot|CO|=|BO|\cdot|OD|$$

818. Determine the infinite nested roots. s=$\sqrt{2\sqrt{4\sqrt{8\sqrt{16\sqrt{\;......}}}}}$

We shall write this as: $\sqrt{2\sqrt{2^2\sqrt{2^3\sqrt{2^4\sqrt{\;......}}}}} \;=\; 2^{\frac{1}{2}}\cdot 2^{\frac{2}{4}}\cdot 2^{\frac{3}{8}}\cdot 2^{\frac{4}{16}}\cdot 2^{\frac{5}{32}}\cdot ..... = 2^{\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+}.....=$

Then we shall look at the sum: $\dfrac{1}{2}+\dfrac{2}{4}+\dfrac{3}{8}+\dfrac{4}{16}+\dfrac{5}{32}+...$ and we split it up in series where the numerator is always 1:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....+$$
$$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+...+$$
$$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+...+$$
$$\frac{1}{16}+\frac{2}{32}+\quad+$$
$$+\frac{1}{32}+\frac{1}{64}\quad+$$

We shall then repeatedly use the formula for the sum of a quotient series:

If $|q|<1$ as in the series above the formula becomes; $\frac{1}{1-q}$, since $|q|^n \to 0 \quad as \;\; n \to \infty$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....+$$
$$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+...+$$
$$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+...+$$
$$\frac{1}{16}+\frac{2}{32}+\quad+$$
$$+\frac{1}{32}+\frac{1}{64}\quad+$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....+=\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right)=1$$
$$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....+=\frac{1}{4}\left(\frac{1}{1-\frac{1}{2}}\right)=\frac{1}{2}$$
$$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....+=\frac{1}{8}\left(\frac{1}{1-\frac{1}{2}}\right)=\frac{1}{4}$$
$$=\frac{1}{16}+\frac{1}{32}+....+=\frac{1}{16}\left(\frac{1}{1-\frac{1}{2}}\right)=\frac{1}{8}$$

So the original sum may be written as:

$$1+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\tfrac{1}{16}+....=1+\tfrac{1}{2}+\left(\tfrac{1}{2}\right)^2+\left(\tfrac{1}{2}\right)^3+\left(\tfrac{1}{2}\right)^4+...=\frac{1}{1-\frac{1}{2}}=2$$

$$\text{So s }=\sqrt{2\sqrt{4\sqrt{8\sqrt{16\sqrt{\;}......}}}}=2^2=4$$

An alternative method is first to evaluate s^2.

$$\left(\sqrt{2\sqrt{4\sqrt{8\sqrt{16\sqrt{\ }......}}}}\right)^2 = \left(2^{\frac{1}{2}} \cdot 2^{\frac{2}{4}} \cdot 2^{\frac{3}{8}} \cdot 2^{\frac{4}{16}} \cdot 2^{\frac{5}{32}} \cdot ..\right)^2 = 2 \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{2}{4}} \cdot 2^{\frac{3}{4}} \cdot 2^{\frac{4}{8}} \cdot 2^{\frac{5}{16}} \cdot$$

$$\frac{2 \cdot 2 \cdot 2^{\frac{3}{2}} \cdot 2^{\frac{3}{4}} \cdot 2^{\frac{4}{8}} \cdot 2^{\frac{5}{16}}}{2^{\frac{3}{2}}} == 2 \cdot 2 \cdot 2^{\frac{3}{2}} \cdot 2^{\frac{3}{4}} \cdot 2^{\frac{4}{8}} \cdot 2^{\frac{5}{16}} =$$

$$2\sqrt{4\sqrt{8\sqrt{16\sqrt{32......}}}} = \sqrt{2^2 \cdot 4\sqrt{4 \cdot \sqrt{16\sqrt{32......}}}} =$$

$$\sqrt{2\sqrt{2^2 4\sqrt{16 \cdot 32\sqrt{32......}}}}$$

$$2\sqrt{2^2\sqrt{2^3\sqrt{2^4\sqrt{2^5......}}}} = 2\sqrt{2\sqrt{2^{3+2}\sqrt{2^4\sqrt{2^5\sqrt{\ }......}}}} =$$

$$2\sqrt{2\sqrt{2^2\sqrt{2^{4+6}\sqrt{2^5\sqrt{\ }.....}}}} = 2\sqrt{2\sqrt{2^3\sqrt{2^{4+6}\sqrt{2^5......}}}} = 2\sqrt{2\sqrt{2^3\sqrt{2^{4+6}\sqrt{2^{8+12}......}}}}$$

816. Solve for *x* : $\ln x^{\ln x} = \ln 10$

$$\ln x^{\ln x} = \ln 10 \quad \Leftrightarrow \quad \ln x \cdot \ln x = \ln 10 \quad \Leftrightarrow \quad (\ln x)^2 = \ln 10 \quad \Leftrightarrow$$

$$\ln x = \pm\sqrt{\ln 10} \quad \Leftrightarrow \quad x = e^{\sqrt{\ln 10}} \quad or \quad x = e^{-\sqrt{\ln 10}}$$

817. Are rectangle is divided vertically into 3 section.

The area of the first section of a rectangle with side length b and height a has area 80, the area of the second section with side length c and height a has area X, and the area of the third section with side length d and height a has area 176.
c+*d*= 40 *and b*+*c* = 32. Determine *X*.

From this we establish the following relations;
$a \cdot b = 80, \quad a \cdot c = X, \quad a \cdot d = 176, \quad b + c = 32, \quad c + d = 40$
$a \cdot b = 80, \ and \ a \cdot d = 176,$

.

From the last two equations we get;
$c + d - (b + c) = 40 - 32 \quad \Rightarrow \quad d - b = 8,$
$ad - ab = a(d - b) = 176 - 80 = 96 \quad \Rightarrow \quad a = \frac{96}{8} = 12$
$ab = 80 \quad \Rightarrow \quad b = \frac{80}{12} = 6\frac{2}{3}, \quad ad = 176 \quad \Rightarrow \quad d = 126\frac{2}{3}$
$b + c = 32 \quad \Rightarrow \quad c = 32 - 6\frac{2}{3} = 25\frac{1}{3}$
$X = ac = 12 \cdot 25\frac{1}{3} = 304$

318. Solve for *x*. $x^{1-\log x} = \frac{1}{100}$

We start by taking the logarithm of both sides:

$\log x^{1-\log x} = \log\frac{1}{100} \quad \Leftrightarrow \quad (1-\log x)\log x = -2 \quad \Leftrightarrow \quad -(\log x)^2 + \log x + 2 = 0$

$d = 1 - 4\cdot(-1)2 = 9; \quad \log x = \dfrac{-1\pm 3}{-210} \qquad \log x = -1 \quad or \quad \log x = 2 \quad \Leftrightarrow \quad x = \dfrac{1}{10} \quad or \quad x = 100$

319. $x^4 + x^3 + x^2 + x + 1 = 0 \quad \Leftrightarrow \quad \dfrac{1-x^5}{1-x} = 0 \qquad no\ real\ solution\ x\ \ is\ diffrent\ \ from\ 1$

But $1 - x^5 \quad \Leftrightarrow \quad x^5 = 1$ has 5 according to Eulers formula: $z^n = w \quad \Leftrightarrow$

$z = \sqrt[5]{|w|}(\cos n\dfrac{2\pi}{5} + i\sin n\dfrac{2\pi}{5}) \quad n = 1..5$

$x^2(x^2+1) + x(x^2+1) + 1 = 0$

$x^3(x+1) + x(x+1) + 1 = 0 \quad \Leftrightarrow \quad (x^3 + x)(x+1) = 1$

$(x^2+1)(x^2+x) + 1 = 0 \quad \Leftrightarrow \quad (x^2+1)x(x+1) + 1 = 0$

320. Which is greater:

$\sqrt{7} - \sqrt{6} \quad or \quad \sqrt{6} - \sqrt{5} \quad we\ \ look\ \ at$

$\sqrt{7} - \sqrt{6} > (\sqrt{6} - \sqrt{5}) \quad \Leftrightarrow \quad \sqrt{7} > 2\sqrt{6} - \sqrt{5} \quad \Leftrightarrow$

$7 > 24 + 5 - 4\sqrt{30} \quad \Leftrightarrow \quad 7 - 29 > -4\sqrt{30} \quad \Leftrightarrow \quad 22 < 4\sqrt{30} \quad \Leftrightarrow$

$444 < 480$

Which is correct

321. In the triangle *ABC*: *C* =2*x*.

The foot point of the height *h* is *H,* The angle *C* is 2*x*. and the angle *ABH* is *x*. |*AH=14*| and |HC|=21. Determine *x*.
We see that the 6angle *HBC* = 90 - 2*x* and the angle *BAH* = 90 – *x*.
We may then establish two expressions for the height:

$h = 14\sin(90-x) \quad and \quad h = 21\sin 2x \quad \Rightarrow \quad 14\sin(90-x) = 21\sin 2x$

$14\cos x = 21\sin x \quad \Rightarrow \quad \sin x = \frac{2}{3} \quad \Rightarrow \quad x = 41^0.81$

322. Determine *b* expressed by *a* by: $\log_{12} 2 = a \quad and \quad \log_6 72 = b$

We have the formula:

$\log_b x = y \quad \Leftrightarrow \quad x = b^y \quad \sin ce \quad \log_b x = \log_b b^y = y\log_b b = y$

So we may write: $\log_{12} 2 = a \quad \Leftrightarrow \quad 2 = 12^a$ And consequently; $\log_6 72 = b \Leftrightarrow \quad 72 = 6^b$

$\frac{72}{2} = \frac{6^b}{12^a} \quad \Leftrightarrow \quad 36 = \frac{6^b}{12^a} \quad \Leftrightarrow \quad \log_6 36 = b - a\log_6 12 \quad \Leftrightarrow$

$\frac{72}{2} = \frac{6^b}{12^a} \quad \Leftrightarrow \quad 36 = \frac{6^b}{12^a} \quad \Leftrightarrow \quad \log_6 36 = b - a\log_6 12 \quad \Leftrightarrow$

$2 = b - a(1+\log_6 2) \quad \Leftrightarrow \quad b = 2 + a(\log_6 2 + 1)$

323. Determine *x* from: $x^x = 2^{3x+192}$

We start with taking log on both sides:

$x\log x = (3x + 2^{192})\log 2$ and we notice that: $2^{192} = 3 \cdot 2^6$,

so 3 is a common factor on the right side, so it must also be on the left side.

We there guess that x is must be a multiply of power 3 of 2. We guess at $x = 2^6$ and then we get:

$2^6 6\log 2 = (3\cdot 2^6 + 3\cdot 2^6)\log 2 \quad \Leftrightarrow \quad 2^7 3\log 2 = 2^6 3\cdot 2\log 2$

So $x = 2^6$ is a soluti0on.

324. determine *x* from: $(i+1)^x = i$

We know that the modulus of a complex number $z = x+iy$ is $(\sqrt{z\cdot\bar{z}} = \sqrt{(x+iy)(x-iy)} = \sqrt{x^2+y^2}$

Since the modulus of the left hand side is $\sqrt{2}^x$ and on the right hand side is 1, x cannot be greater than 1. We take the natural logarithm on both sides:

The logarithm of a complex number $\ln z = \ln(|z|e^{i\varphi}) = \ln(|z| + i(\varphi + 2p\pi) \quad p = 1...n...$

We get the equation; $\ln(\ln(i+1)^x = \ln i \quad \Leftrightarrow \quad x\ln(i+1) = \ln i$

$\ln i = 1\cdot i(\frac{\pi}{2} + p2\pi) \quad and \quad \ln(i+1) = \sqrt{2} + i(\frac{\pi}{4} + p2\pi)$ we get:

$(i+1)^x = i \quad \Leftrightarrow \quad x\ln(1+i) = \ln i \quad \Leftrightarrow \quad x = \frac{\ln i}{\ln i + 1)} = \frac{1 + i(\frac{\pi}{2} + p2\pi)}{= \sqrt{2} + i(\frac{\pi}{4} + p2\pi)}.$