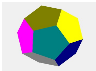
Gain probabilities
in
EuroJackpot and Lotto



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1. Introduction

Some years ago a new lotto game was introduced in Denmark and in the rest of Europe.
One could read on the posters that on the average, there would be a gain on every twentieth row.
And why should it not be true, since at least before 1988, some of the students in the last year in the Danish 9-12 grade high school, having mathematics at the highest level, would be able to verify the assertion.
I tried the game (for fun), but after making 4 rows for 5 weeks, without a gain, I decided to find out, whether the assertion was correct.

Since the foundation of combinatorics and probability are no longer a part of the Danish 9-12 year curriculum in mathematics, I shall briefly outline the theoretic foundation of these subjects.

2. Fundamentals of probability

If an outcome space in an "experiment" has n elements (a n set) then the number of different ways to select a q-subset is given by the formula:

$$C(n,q)=\frac{n!}{(n-q)!q!}$$

Where the factorial $n!$as usual denotes the product: $1\cdot 2\cdot 3\cdots(n-1)n$

An *event H* is defined as a *subset* in the outcome space *U*.

In a symmetric outcome field, where all outcomes share the same probability, so the probability of an outcome is $1/n$, since the sum of probabilities for all outcomes must add up to 1.

If we denote the number of elements in an event by $n(H)$ then the probability of that event, can then be calculated by the formula:

$$P(H)=\frac{n(H)}{n(U)}=\frac{\text{number of elements in } H}{\text{number of elements in } U}$$

(The outcome space U and the empty set $Ø$ are also events, with $n(U) = n$ and $n(Ø) = 0$)

3. EuroJackpot

A row in EuroJackpot consists of one row with 5 fields, which must be filled out with one of the numbers 1...50 and one row having 2 fields, which must be filled out with the numbers1..8.
We shall now calculate the probability of each of the events which results in a gain.

1. First prize is: 5+2 correct numbers. The 5 numbers may be chosen in $C(50,5)$ different ways and the two alternative numbers in $C(8,2)$ different ways.
   This results in $C(50,5)\cdot C(8,2) = 59.325.280$ possible outcomes.
   The possibility of getting the jackpot is therefore:

$$P(5+2) = \frac{1}{59{,}325{,}280} = 1.686\ 10^{-8}$$

2. The second prize is: 5+1 correct numbers: The correct number may be chosen in two ways among the two add numbers, the other may be chosen among 6 numbers, that are not a winning number.

$$P(5+1) = 2\cdot 6\cdot P(5+2) = 12\cdot P(5+2) = 2.022\ 10^{-7}$$

3. The third prize: 5 correct numbers:. The two add numbers may be chosen among the 6 numbers that is not a winning number in $C(6,2) = 15$ different ways.

$$P(5) = 15\cdot P(5+2) = 2.528\cdot 10^{-7}$$

4. The 4th prize: 4+2 correct numbers: The 4 correct numbers may be chosen among the 5 winning numbers $C(5,4) = 5$ different ways, and the last number may be chosen in $C(45,1) = 45$ different ways. In all $5\cdot 45 = 225$ different ways.

$$P(4+2) = 225\cdot P(5+2) = 3.793\cdot 10^{-6}$$

5. The 5th prize: 4+1 correct numbers: The number of possibilities are: $K(5,4)\cdot K(45,1)\cdot 2\cdot 6 = 2700$

$$P(4+1) = 2700\cdot P(5+2) = 4.552\cdot 10^{-5}$$

6. The 6th prize: 4 correct numbers:
   The number of possibilities are: $C(5,4)\cdot C(45,1)\cdot C(6,2) = 3375$.

$$P(4) = 3375\cdot P(5+2) = 5.689\cdot 10^{-5}$$

7. The 7th prize: 3+2 correct numbers: The number of possibilities are: $C(5,3)\cdot C(45,2) = 9900$.

$$P(3+2) = 9900\cdot P(5+2) = 1.688\cdot 10^{-4}$$

8. The 8th prize: 3+1 correct numbers: The number of possibilities are: $C(5,3)\cdot C(45,2)\cdot 2\cdot 6 = 118.800$

$$P(3+1) = 118.800\cdot P(5+2) = 2.003\cdot 10^{-3}$$

9. The 9th prize: 2+2 correct numbers: The number of possibilities are: $C(5,2)\cdot C(45,3) = 141,900$.

$$P(2+2) = 141.900\cdot P(5+2) = 2.392\cdot 10^{-3}$$

10. The 10th prize: 3 correct numbers: The number of possibilities are: $C(5,3)\cdot C(45,2)\cdot C(6,2) = 148,500$.

$$P(3) = 148,500\cdot P(5+2) = 2.503\cdot 10^{-3}$$

11. The 11th: prize: 1+2 correct numbers: The number of possibilities are: $C(5,1)\cdot C(45,4) = 744,975$.

$$P(1+2) = 744,975\ P(5+2) = 1,256\cdot 10^{-2}$$

12. The 12th prize: 2+1 correct numbers: The number of possibilities are:
    $C\,(5,2)\cdot C\,(45,3)\cdot 6\cdot 2 = 1{,}702{,}800$.

$$P\,(2+1) = 1.702.800\cdot P\,(5+2) = 0.02871$$

4. The mean gain

If you want to calculate the mean gain, it can be done by adding all the possibilities, but it is easier to add up the number of outcomes of all the disjoint events that results in a gain, giving 2,873,203 possibilities. Multiplying this number by $P\,(5+2)$, we find the possibility of a gain.

$$\text{P(Gain in EuroJackpot)} = 0.04843.$$

Since this number is close to 5%, one may concede, that on the average the player may have a gain on every 20th row.
That Eurojackpot is far from being a profitable game in the long run is obvious, but we may estimate the mean gain in one row. However the numbers below are in Danish crowns, and one US-dollar is roughly 7 crowns (kr.).
One row costs 15 crowns, We put the jackpot gain to 63,000,000 crowns, and if we use the average gains for one week, we may calculate the statistical mean: G denotes gain.

$$E(X) = G(5+2)P(5+2) + G(5+1)P(5+1) +\ldots+ G(2+1)P(2+1) = 4.20 \text{ kr.}$$

The mean of the gain is therefore estimated to 4.20 - 15 kr. = -11.80 kr.

This is however a much poorer result, compared to e.g. roulette or blackjack

5. Danish Lotto game

In the Danish lotto system are drawn 7 numbers and an add number from the 36 numbers 1,2,3,…,36.

For the 7 ordinary numbers this results in $C(36,7) = 8{,}347{,}680$ different possibilities.

1. First prize: 7 correct numbers: $P(7) = \dfrac{1}{C(36,7)} = 1.1979\cdot 10^{-7}$

2. The 2th prize: 6 correct numbers plus the add number: The 6 correct numbers may be chosen among 7 correct numbers in $C(7,6) = 7$ different ways.

$$P(6+1) = 7\cdot P(7) = 8.385\ 10^{-7}$$

3. The 2th prize: 6 correct numbers. The 6 correct numbers may be chosen among the 7 correct numbers in $C(7,6) = 7$ different ways, and the last number may be chosen in 36-7-1 =28 different ways, which gives K(7,6)·28 = 196 different ways.

$$P(6) = 196\cdot P\,(7) = 2.358\ 10^{-5}$$

4. The 2th prize: 5 correct numbers. These may be chosen in C (7,5) different ways and the last two numbers may be chosen among the 36-7 = 29 numbers in C (29,2) different ways, resulting in K(7,5) · K(29,2) = 8526 different ways.

$$P(5) = 8526 \cdot P\,(7) = 1.021\ 10^{-3}$$

5. The 3th prize: 4 correct numbers: K(7,4) ·K(29,3) = 127.890

$$P(4) = 127.890 \cdot P\,(7) = 0.01532 = 1{,}53\%$$

Thus the number of possibilities for any gain are 136.620, which gives the possibility for a gain on:

$$P\,(\text{gain}) = 136{,}620 \cdot P\,(7) = 0.01637 = 1.64\%$$

At a glance it looks like that the gain result is better in EuroJackpot, than in lotto, but you consider that a EuroJackpot row costs 15 kr. whereas a lotto row costs 5 kr. Since 15/5 1.64% = 4.92% > 4,8% , it is not necessarily the case.

You may find a comprehensive treatment on probabilities and strategies for games in my home page: www.olewitthansen.dk