Uden tilnærmelserne $\cos(\theta-\varphi)\approx 1 \quad og \quad \sin(\theta-\varphi)\approx \theta-\varphi$ bliver regningerne kun lidt vanskeligere

Ligning for θ:

$$l_1^{\,2}(m_1+m_2)\ddot{\theta}+m_2l_1l_2\ddot{\varphi}\cos(\theta-\varphi)-m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+(m_1+m_2)gl_1\sin\theta+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

Ligning for φ

$$m_2l_2^{\,2}\ddot{\varphi}+m_2l_1l_2\ddot{\theta}\cos(\theta-\varphi)-m_2l_1l_2\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+m_2gl_2\sin\varphi+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

Først isolerer vi θ'':

Vi ganger den første ligning med $m_2l_2^{\,2}$ og den anden ligning med $-m_2l_1l_2\cos(\theta-\varphi)$ og adderer ligningerne

$$l_1^{\,2}(m_1+m_2)\ddot{\theta}+m_2l_1l_2\ddot{\varphi}\cos(\theta-\varphi)-m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+(m_1+m_2)gl_1\sin\theta+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

I: $m_2l_2^{\,2}l_1^{\,2}(m_1+m_2)\ddot{\theta}+m_2l_2^{\,2}m_2l_1l_2\ddot{\varphi}\cos(\theta-\varphi)-m_2l_2^{\,2}m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+$
$m_2l_2^{\,2}(m_1+m_2)gl_1\sin\theta+m_2l_2^{\,2}m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$

$$m_2l_2^{\,2}\ddot{\varphi}+m_2l_1l_2\ddot{\theta}\cos(\theta-\varphi)-m_2l_1l_2\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+m_2gl_2\sin\varphi+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

II:

$$-m_2l_1l_2\cos(\theta-\varphi)m_2l_2^{\,2}\ddot{\varphi}-m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\ddot{\theta}\cos(\theta-\varphi)+m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})-$$
$$m_2l_1l_2\cos(\theta-\varphi)m_2gl_2\sin\varphi-m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

I+II

$$m_2l_2^{\,2}l_1^{\,2}(m_1+m_2)\ddot{\theta}-m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\ddot{\theta}\cos(\theta-\varphi)-m_2l_2^{\,2}m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+$$
$$m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+m_2l_2^{\,2}(m_1+m_2)gl_1\sin\theta-m_2l_1l_2\cos(\theta-\varphi)m_2gl_2\sin\varphi+$$
$$m_2l_2^{\,2}m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)-m_2l_1l_2\cos(\theta-\varphi)m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

Ved division med $m_2l_1l_2$ og samle leddene fås:
I+II: θ''

$$(l_2l_1((m_1+m_2)-m_2\cos(\theta-\varphi)^2))\ddot{\theta}-m_2l_2\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})_2(l_2\dot{\varphi}-l_1\cos(\theta-\varphi)\dot{\theta})+$$
$$gl_2((m_1+m_2)\sin\theta-\cos(\theta-\varphi)m_2\sin\varphi)+(m_2l_2^{\,2}\sin(\theta-\varphi)-\cos(\theta-\varphi)\sin(\theta-\varphi))\dot{\theta}\dot{\varphi}=0$$

Vi indfører derefter konstanter: a_1, a_2,…

I+II: Løst for θ''

$$a_1((m_1+m_2)-m_2\cos(\theta-\varphi)^2)\ddot{\theta}-a_3\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})(l_2\dot{\varphi}-l_1\cos(\theta-\varphi)\dot{\theta})+$$
$$gl_2((m_1+m_2)\sin\theta-\cos(\theta-\varphi)m_2\sin\varphi)+(a_4\sin(\theta-\varphi)-\cos(\theta-\varphi)\sin(\theta-\varphi))\dot{\theta}\dot{\varphi}=0$$

$l_1=0{,}50\ m, l_2=0{,}20\ m, m_1=0{,}10\ kg, m_2=0{,}030\ kg$
$a_1=l_2l_1=0{,}1,\ \ a_3=m_2l_2=0{,}006,\ \ a_4=m_2l_2^{\ 2}=0{,}0012,\ \ a_5=gl_2=1{,}96$

Vi indsætter konstanterne.

I+II: θ med konstanter:
$$0{,}1(0{,}13-0{,}03\cos(\theta-\varphi)^2)\ddot{\theta}-0{,}006\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})(0{,}20\dot{\varphi}-0{,}50\cos(\theta-\varphi)\dot{\theta})+$$
$$1{,}96(0{,}13\sin\theta-0{,}03\cos(\theta-\varphi)\sin\varphi)+(0{,}0012\sin(\theta-\varphi)-\cos(\theta-\varphi)\sin(\theta-\varphi))\dot{\theta}\dot{\varphi}=0$$

Så isolerer vi fi''

$$l_1^{\ 2}(m_1+m_2)\ddot{\theta}+m_2l_1l_2\ddot{\varphi}\cos(\theta-\varphi)-m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+(m_1+m_2)gl_1\sin\theta+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

$$m_2l_2^{\ 2}\ddot{\varphi}+m_2l_1l_2\ddot{\theta}\cos(\theta-\varphi)-m_2l_1l_2\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+m_2gl_2\sin\varphi+m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

Vi ganger den øverste ligning med $m_2l_1l_2\cos(\theta-\varphi)$ og den nederste med $-l_1^{\ 2}(m_1+m_2)$ Og divideres igennem med $m_2l_1l_2$

I: $\cos(\theta-\varphi)l_1^{\ 2}(m_1+m_2)\ddot{\theta}+\cos(\theta-\varphi)m_2l_1l_2\ddot{\varphi}\cos(\theta-\varphi)-\cos(\theta-\varphi)m_2l_1l_2\dot{\varphi}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})+$
$\cos(\theta-\varphi)(m_1+m_2)gl_1\sin\theta+\cos(\theta-\varphi)m_2l_1l_2\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$

II: $-l_1l_2(m_1+m_2)\ddot{\varphi}-l_1^{\ 2}(m_1+m_2)\ddot{\theta}\cos(\theta-\varphi)+l_1^{\ 2}(m_1+m_2)\dot{\theta}\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})-$
$l_1(m_1+m_2)g\sin\varphi-l_1^{\ 2}(m_1+m_2)\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$

I+II: φ ''
$$b_1(m_2\cos(\theta-\varphi)^2-(m_1+m_2))\ddot{\varphi}+\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})(b_2\dot{\theta}-\cos(\theta-\varphi)b_3\dot{\varphi})+$$
$$b_4(\cos(\theta-\varphi)\sin\theta-\sin\varphi)+(b_5\cos(\theta-\varphi)-b_6)\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$

Vi indfører derefter konstanter: $b_1, b_2,\ldots$
$b_1=l_1l_2=0{,}10,\ \ b_2=l_1^{\ 2}(m_1+m_2)=0{,}0325,\ \ b_3=m_2l_1l_2=0{,}003,\ \ b_4=(m_1+m_2)gl_1=0{,}638,$
$b_5=m_2l_1l_2=0{,}003,\ \ b_6=l_1^{\ 2}(m_1+m_2)=0{,}0325$

$l_1=0{,}50\ m, l_2=0{,}20\ m, m_1=0{,}10\ kg, m_2=0{,}030\ kg$

I+II: φ '' med konstanter

$$0{,}10(0{,}03\cos(\theta-\varphi)^2-0{,}13)\ddot{\varphi}+\sin(\theta-\varphi)(\dot{\theta}-\dot{\varphi})(0{,}0325\dot{\theta}-\cos(\theta-\varphi)0{,}003\dot{\varphi})+$$
$$0{,}638(\cos(\theta-\varphi)\sin\theta-\sin\varphi)+(0{,}003\cos(\theta-\varphi)-0{,}0325)\dot{\theta}\dot{\varphi}\sin(\theta-\varphi)=0$$